Tender Heart High School,Sector 33B,Chd.

Class9th Mathematics Date 8.7.2024

Chapter 4 Factorisation

FACTORISATION

→ splitting of a number into its factors or divisors is called factorisation. e.g. $12 = 3 \times 4$ or $2 \times 2 \times 3$

Example 1

Factorisation

$$(x-2)(x+2) = x^2 - 4$$

Expansion

Example 2

Factorisation

$$(2x+3)(x-1) = 2x^2 + x - 3$$

Expansion

So, factorisation is 'splitting' an expression into a multiplication of simpler expression. It is the reverse process of multiplication.

→ Now, we discuss different techniques of factorisation.

I Factorisation by Taking Out the Common Factor

Examples (i) $6p + 8q = 2(3p + 4q)$

(ii) $20x^4y - 12x^3y^3 = 4x^2y(5x^2 - 3xy^2)$

(iii) $x(a+b) + y(a+b) = (a+b)(x+y)$

II Factorisation by Grouping

Examples (i) $a^2 + bc + ab + ac = (a^2 + ab) + (bc + ac)$

$= a(a+b) + c(b+a) = (a+b)(a+c)$

(ii) $6ab - b^2 + 12ac - 2bc = (6ab + 12ac) - (b^2 + 2bc)$

$= 6a(b+2c) - b(b+2c) = (b+2c)(6a-b)$

Note: We can solve this by taking different groups also i.e. $6ab - b^2 + 12ac - 2bc = (6ab - b^2) + (12ac - 2bc)$

$= b(6a - b) + 2c(6a - b) = (6a-b)(b+2c)$

Rules:-

1) Arrange the terms of the given polynomial in groups. in such a way that each group has a common factor.
2) Factorise each group.
3) Take out the factor which is common to each group.

Ms. Reena Tyagi

Class 9th Maths.

III Difference of two squares $a^2-b^2=(a+b)(a-b)$

Examples (i) $25a^2-9b^2=(5a)^2-(3b)^2=(5a+3b)(5a-3b)$

(ii) $2x^3-50x=2x(x^2-25)=2x(x^2-5^2)=2x(x+5)(x-5)$

(iii) $1-(b-c)^2=(1)^2-(b-c)^2=[1+(b-c)][1-(b-c)]$
$=(1+b-c)(1-b+c)$

(iv) $x^4-81=x^4-3^4=(x^2)^2-(3^2)^2=(x^2)^2-(9)^2$
$=(x^2+9)(x^2-9)=(x^2+9)(x+3)(x-3)$

Note: Here, identity is applied two times.

(v) $a^2+b^2-c^2-2ab=(a^2+b^2-2ab)-c^2=(a-b)^2-c^2$
$=[(a-b)+c][(a-b)-c]=(a-b+c)(a-b-c)$

IV Sum and Difference of Two Cubes

$a^3+b^3=(a+b)(a^2-ab+b^2)$
$a^3-b^3=(a-b)(a^2+ab+b^2)$

[Derivation is given in book at page no.(3.13)]

Examples:-

(i) $8a^3+125b^3=(2a)^3+(5b)^3=(2a+5b)((2a)^2+2a\times 5b+(5b)^2)$
$=(2a+5b)(4a^2-10ab+25b^2)$

(ii) $x^3-343=x^3-7^3=(x-7)(x^2+x\times 7+7^2)$
$=(x-7)(x^2+7x+49)$

(iii) $27x^3-\frac{125}{x^3}=(3x)^3-\left(\frac{5}{x}\right)^3=\left(3x-\frac{5}{x}\right)\left[(3x)^2+3x\times\frac{5}{x}+\left(\frac{5}{x}\right)^2\right]$
$=\left(3x-\frac{5}{x}\right)\left(9x^2+15+\frac{25}{x^2}\right)$

V Factorisation of a trinomial by splitting the middle term :-

Consider polynomial of the form ax^2+bx+c
where, a, b and c are constants

→ coefficient of x^2 is 'a', coefficient of x is 'b' and constant is 'c'

→ Find the product of 'a' and 'c' $=ac$

→ split 'b' into 2 parts 'p' and 'q', such that
$p+q=b$ and $pq=ac$

→ Write coefficient of x i.e. 'b' as $(p+q)$

→ Factorize by grouping terms.

Now, let us consider the following examples :-

1) $6x^2 + 17x + 5$

Here, we need to find two numbers whose sum = 17 and whose product = $5 \times 6 = 30$

Numbers	Sum	Product	
6 and 5	11	30	×
3 and 10	13	30	×
2 and 15	17	30	✓

From the table we can see that there are different combination of numbers, but here we consider numbers whose sum is 17 i.e. 2 and 15

$\Rightarrow 6x^2 + 17x + 5 = 6x^2 + 2x + 15x + 5 = 2x(3x+1) + 5(3x+1)$

$= (3x+1)(2x+5)$

2) $3x^2 - x - 4$

Here, we need to find two numbers whose sum is -1 and whose product = $-4 \times 3 = -12$

Numbers	Sum	Product	
−4 and 3	−1	−12	✓
4 and −3	1	−12	×
−6 and 2	−4	−12	×
6 and −2	4	−12	×

so, we consider -4 and 3 which gives the sum as -1

$\Rightarrow 3x^2 - x - 4 = 3x^2 - 4x + 3x - 4 = x(3x-4) + 1(3x-4)$

$= (3x-4)(x+1)$

3) $3(a-2b)^2 - 2(a-2b) - 8$

Put $a - 2b = x$, we get

$3x^2 - 2x - 8$

Here, we need to find two numbers whose sum is -2 and whose product = $-8 \times 3 = -24$

Numbers	Sum	Product	
−8 and 3	−5	−24	×
8 and −3	5	−24	×
−6 and 4	−2	−24	✓
6 and −4	2	−24	×
...			

so, we consider 4 and -6 which gives the sum as -2

$\Rightarrow 3x^2 - 2x - 8 = 3x^2 - 6x + 4x - 8 = 3x(x-2) + 4(x-2)$

$= (x-2)(3x+4) = (a-2b-2)(3a-6b+4)$

Derivations

Using the expansion $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

We have

$$
\begin{aligned}
a^3 + b^3 &= (a + b)^3 - 3a^2b - 3ab^2 \\
&= (a + b)^3 - 3ab(a + b) \\
&= (a + b)\,\{(a + b)^2 - 3ab\} \\
&= (a + b)\,\{a^2 + 2ab + b^2 - 3ab\} \\
&= (a + b)\,(a^2 - ab + b^2)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
a^3 - b^3 &= (a - b)^3 + 3a^2b - 3ab^2 \\
&= (a - b)^3 + 3ab\,(a - b) \\
&= (a - b)\,\{(a - b)^2 + 3ab\} \\
&= (a - b)\,\{a^2 - 2ab + b^2 + 3ab\} \\
&= (a - b)\,(a^2 + ab + b^2)
\end{aligned}
$$