TENDER HEART HIGH SCHOOL, Sec. 33B, CHD.

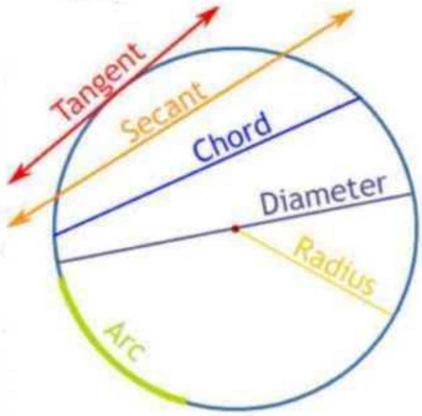
Subject: Mathematics Date: 13.5.2024

Class: 9 Teacher: Ms. Reena

Chapter-13

Chord Properties of a circle
&
Arc Properties of a circle

What is a Circle?



A circle is a closed plane curve such that each point on it is at a constant distance from a fixed point lying on the same plane.

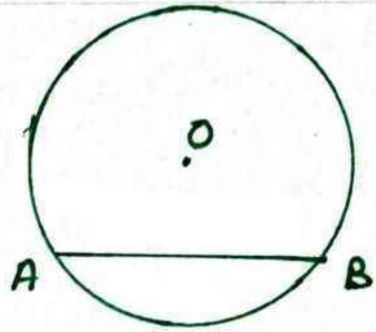
A circle is also defined as the locus of a point which moves in a plane is such a way that it is always at a constant distance from a fixed point lying on the same plane.

The fixed point is known as the centre of the circle, and the constant distance between the centre and any point on the circle is known as the radius of the circle.

If a point P, lying on a circle, moves along the circle, completes one full rotation, and return to its original position then the distance covered by it is known as the circumferece.

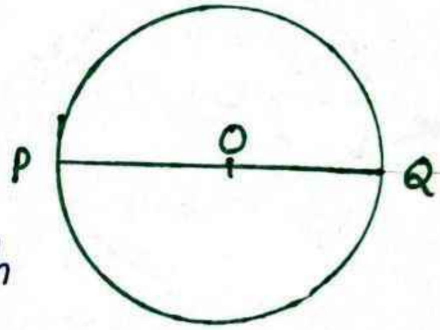
Chord

The line segment joining any two points on a circle is called a chord of the circle.
Here, AB is a chord.



Diameter

A chord passing through the centre of a circle is called a diameter. The length of a diameter is twice the length of the radius of the circle.

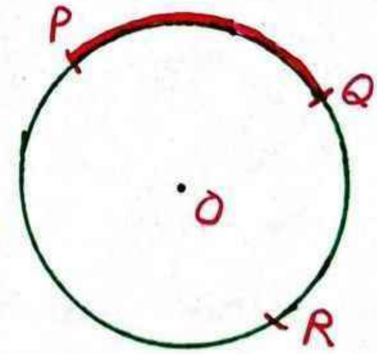


Here, PQ is a diameter and PQ = 2 × OP

A diameter is the largest chord of a circle.

Arc

A (continuous) part of a circle is called an arc of the circle.
The arc of a circle is denoted by the symbol " $\frown$ "

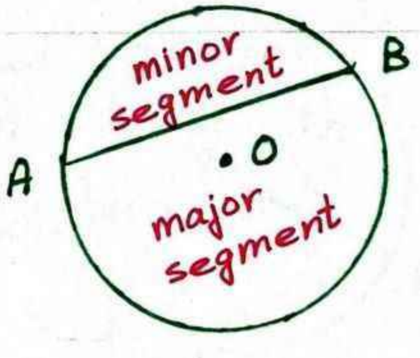


In the adjoining figure, $\overset{\frown}{PQ}$ denotes the arc PQ of the circle with centre O.

Generally, a chord divides the circumference of a circle into two unequal parts. The greater one is known as the major arc and the shorter one is known as the minor arc. $\overset{\frown}{PQ}$ is minor arc, $\overset{\frown}{PRQ}$ is major arc.

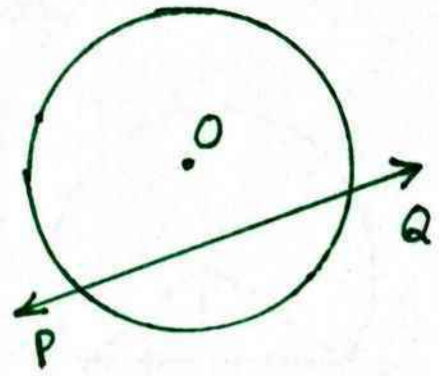
Segment

A chord of a circle divides its circular region into two parts. Each part of the circular region is called a segment of the circle.



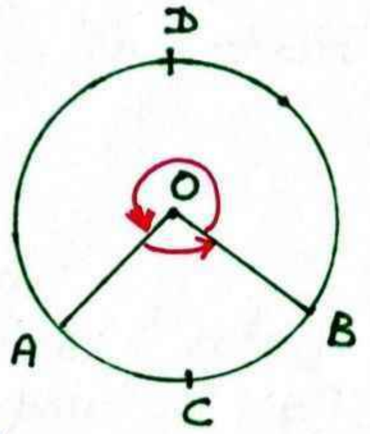
Secant

A line which meets a circle in two points is called a secant of the circle.
Here, PQ is a secant of the circle with centre O.



Angle at the centre

If two radii are drawn to join the extremities of an arc to the centre of a circle, then the angle formed between those two radii is known as the angle subtended by the arc at the centre of the circle.

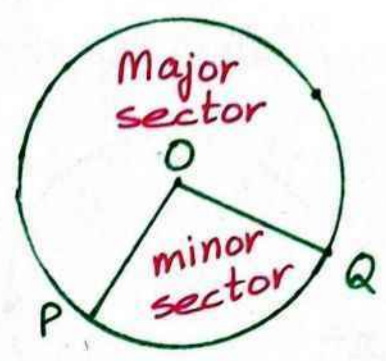


Here, $\overset{\frown}{ACB}$ subtends $\angle AOB$ at the centre and $\overset{\frown}{BDA}$ subtends reflex $\angle AOB$ at the centre.

Sector

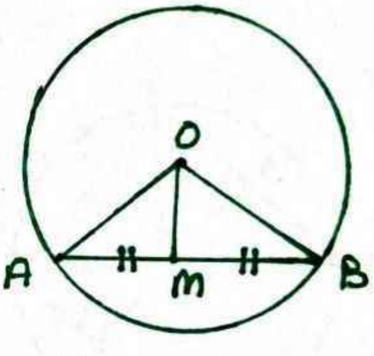
The part of the plane region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.

The part containing the minor arc is called minor sector and the part containing the major arc is called major sector.



CHORD PROPERTIES OF CIRCLES

Theorem 1:- The straight line drawn from the centre of a circle to bisect a chord, which is not a diameter, is perpendicular to the chord.



Given:- A chord AB of a circle with centre O, and OM bisects the chord AB.

To prove:- $OM \perp AB$

Construction:- Join OA and OB

Proof:- In $\triangle OAM$ and $\triangle OBM$

$OA = OB$ [radii of same circle]

$AM = MB$ [M is mid-point of AB]

$OM = OM$ [Common]

$\triangle OAM \cong \triangle OBM$ [S.S.S. rule of congruency]

$\angle AMO = \angle OMB$ [c.p.c.t]

$\angle AMO + \angle OMB = 180^\circ$ [AMB is a straight line]

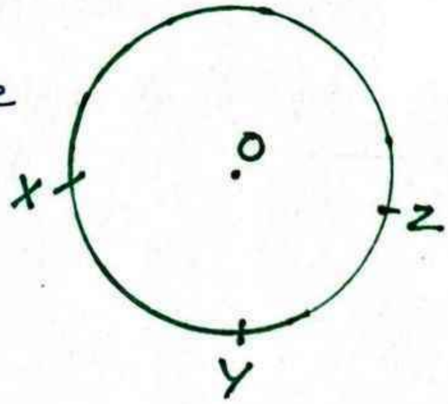
$\angle AMO = 90^\circ$ [$\because \angle AMO = \angle OMB$]

Hence $OM \perp AB$

Theorem 2:- (Converse of theorem 1)

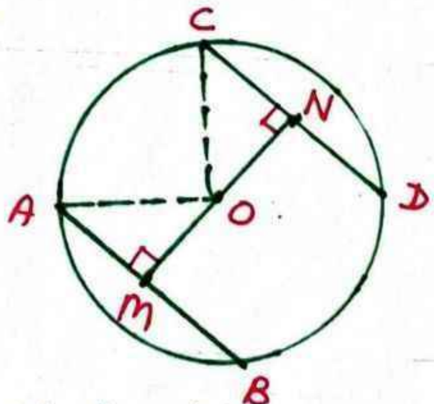
The perpendicular to a chord from the centre of the circle bisects the chord.

Theorem 3:- There is one and only one circle which passes through three given points not in a straight line.



Here, X, Y and Z are the three non collinear points, that is they are not in a straight line.

Theorem 4:- Equal chords of a circle are equidistant from the centre.



Given:- AB and CD are chords of a circle with centre O, and AB = CD

To Prove:- AB and CD are equidistant from O that is if $OM \perp AB$ and $ON \perp CD$, then OM = ON

Construction:- Join OA and OC

Proof:- We know that perpendicular from centre bisects the chord

$\Rightarrow AM = \frac{1}{2} AB$ and $CN = \frac{1}{2} CD$

$\Rightarrow AM = CN$ [AB = CD (given)]

Now, In ΔOAM and ΔOCN

$AM = CN$ [proved above]

$\angle AMO = \angle CNO$ [each = 90°, $OM \perp AB$ and $ON \perp CD$]

$OA = OC$ [Radii of same circle]

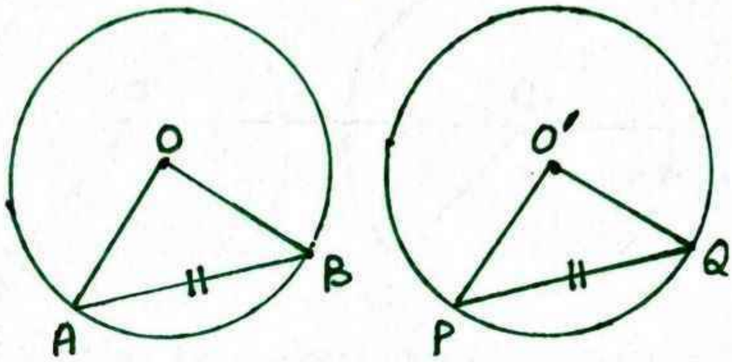
$\therefore \Delta OAM \cong \Delta OCN$ [R.H.S. rule of congruency]

$OM = ON$ [c.p.c.t.]

Class 9^{th}, Mathematics

CHORDS AND ARCS PROPERTIES

Theorem 7:- In equal circles (or in the same circle), equal chords cut off equal arcs.



Given :- AB and PQ are chords of two equal circles with centres O and O' and AB = PQ

To Prove :- Arc AB = Arc PQ

Construction:- Join OA, OB, O'P and O'Q

Proof:- In ΔOAB and $\Delta O'PQ$

$OA = O'P$ [Radii of equal circles]

$OB = O'Q$

$AB = PQ$ [Given]

$\therefore \Delta OAB \cong \Delta O'PQ$ [S.S.S. rule of congruency]

$\angle AOB = \angle PO'Q$ [c.p.c.t.]

$\widehat{AB} = \widehat{PQ}$

Conversely, In equal circles (or in same circle), if two arcs are equal then their chords are equal.

that is if arc $\widehat{AB}$ = arc $\widehat{PQ}$ then $AB = PQ$

Equal arcs (congruent arcs)

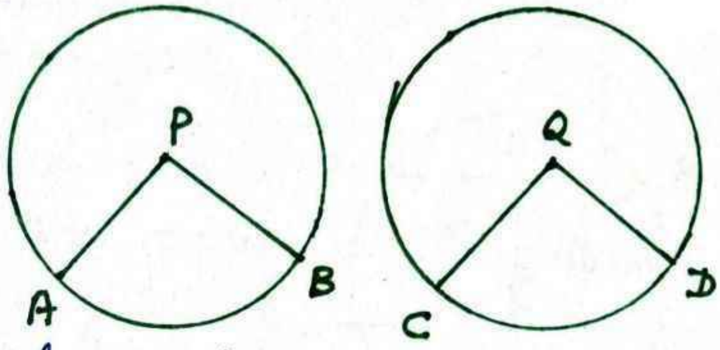
Two arcs of the same circle (or congruent circles) having equal measures are called equal arcs (congruent arcs)

Theorem 6:- In equal circles (or in the same circle), if two arcs subtend equal angles at the centre, they are equal.

that is

If $\angle APB = \angle CQD$

then $\widehat{AB} = \widehat{CD}$

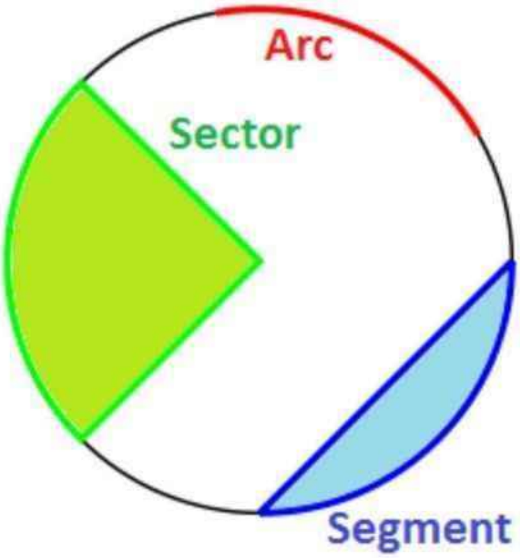


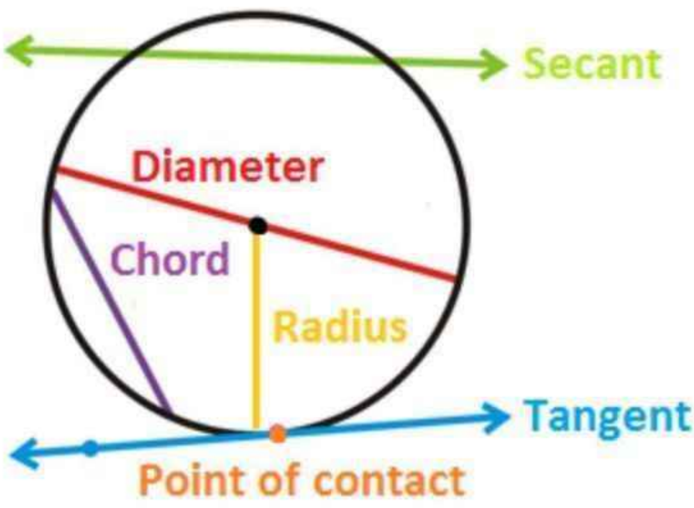
Conversely,

If two arcs of equal circles (or in the same circle) are equal, they subtend equal angles at the centre.

So, in same circle (or of equal circles)

1) Equal chords subtend equal angles at the centre of the circle.
2) Equal angles at the centres make equal chords.





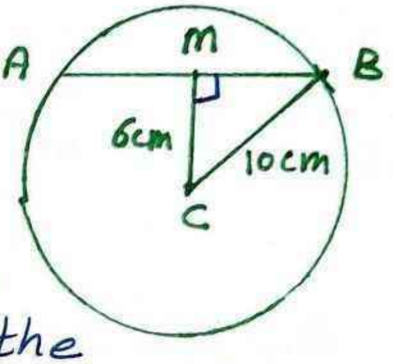
TENDER HEART SCHOOL, Sec. 33B, CHD.

Class : 10^{th}

Subject : Mathematics

Ms. Reena

Example 1 Let AB be a chord of a circle with centre M and radius = 10cm.



Distance from centre to chord = 6cm

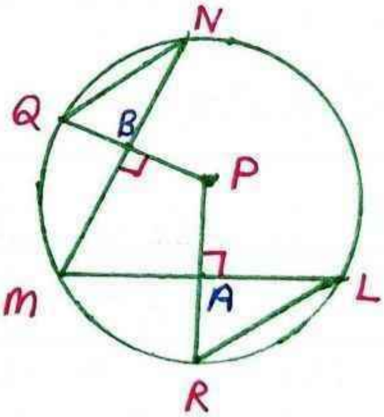
We know that perpendicular to the chord from the centre bisects the chord,

So, $MB = \frac{1}{2}AB$

Now, from right-angled ΔBCM, we have

$$MB = \sqrt{(10)^2 - (6)^2} = \sqrt{100-36} = \sqrt{64} = 8cm$$

Example 2 Given : MN and ML are chords of a circle with centre P and $MN = ML$ and $PQ \perp MN$ and $PR \perp ML$



QN and RL are joined

To Prove :- $QN = RL$

Proof :- $PR \perp ML \Rightarrow A$ is the mid point of ML

Therefore, $MA = AL \Rightarrow AL = \frac{1}{2}ML$

Similarly, $PQ \perp MN \Rightarrow BN = \frac{1}{2}MN$

But $ML = MN \Rightarrow \underbrace{AL = BN}_{(i)}$ [$\because$ chord ML = chord MN]

Also, equal chords are equidistant from the centre $\Rightarrow PB = PA$

Now, $PQ = PR$ [both are radii]

Since $PA = PB$ and $PR = PQ$

$\Rightarrow AR = BQ$ ------ (ii)

Now, In ΔLRA and ΔNQB

$AR = BQ$ [From (ii)]

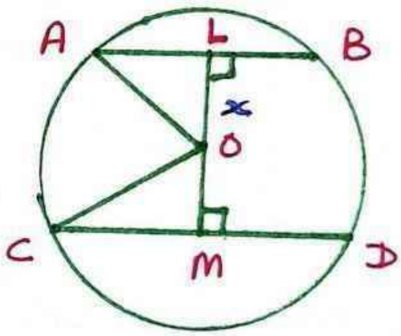
$BN = AL$ [From (i)]

$\angle RAL = \angle QBN = [90° \text{ each}]$

Therefore, $\Delta LRA \cong \Delta NQB$ [SAS axiom of congruency]

So, $QN = RL$ (c.p.c.t.)

Example 3 Let O be the centre of the circle and let its radius be r cm.



Draw $OL \perp AB$ and $OM \perp CD$

Then, $AL = \frac{1}{2} AB = 5\text{cm}$ and

$CM = \frac{1}{2} CD = 12\text{cm}$

Since $AB \parallel CD$, then points O, L, M are collinear and therefore, $LM = 17\text{cm}$

Let $OL = x$ cm. Then, $OM = (17-x)$ cm

Join OA and OC. Then $OA = OC = r$ cm

Now, from right-angled ΔOLA and ΔOMC,

we have $(OA)^2 = (OL)^2 + (AL)^2$ [By Pythagoras Th.]

and $(OC)^2 = (OM)^2 + (CM)^2$

$\Rightarrow r^2 = x^2 + (5)^2$ ------ (i)

$r^2 = (17-x)^2 + (12)^2$ ------ (ii)

$\Rightarrow x^2 + (5)^2 = (17-x)^2 + (12)^2$ [From (i) and (ii)]

$\Rightarrow x^2 + 25 = x^2 - 34x + 433$

$\Rightarrow 34x = 408$ $\Rightarrow x = 12$

On substituting $x = 12$ in (i), we get $r = 13$ cm