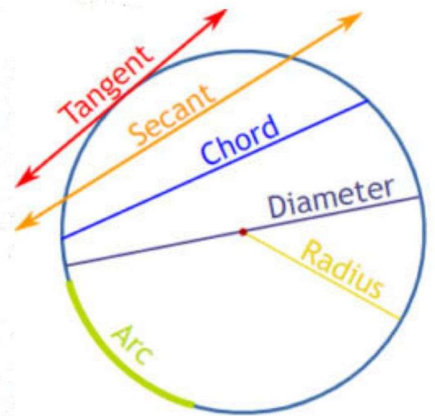


### Chapter-13

## Chord Properties of a circle & Arc Properties of a circle

### What is a Circle ?

A circle is a closed plane curve such that each point on it is at a constant distance from a fixed point lying on the same plane.



A circle is also defined as the locus of a point which moves in a plane in such a way that it is always at a constant distance from a fixed point lying on the same plane.

The fixed point is known as the centre of the circle, and the constant distance between the centre and any point on the circle is known as the radius of the circle.

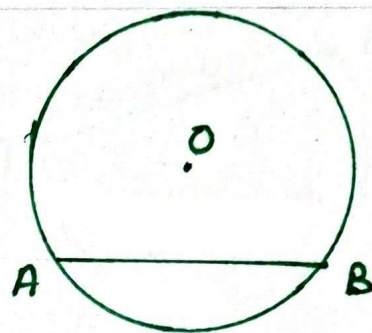
If a point P, lying on a circle, moves along the circle, completes one full rotation, and return to its original position then the distance covered by it is known as the circumference.



Chord

The line segment joining any two points on a circle is called a chord of the circle.

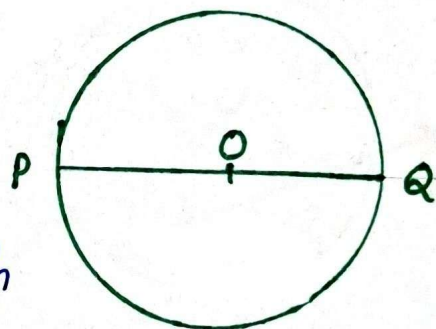
Here, AB is a chord.

Diameter

A chord passing through the centre of a circle is called a diameter. The length of a diameter is twice the length of the radius of the circle.

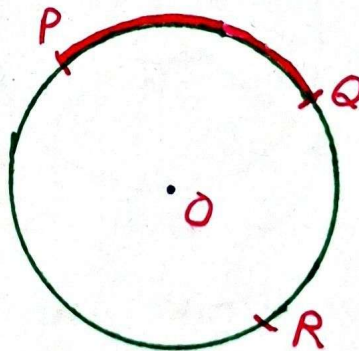
Here, PQ is a diameter and  $PQ = 2 \times OP$

A diameter is the largest chord of a circle.

Arc

A (continuous) part of a circle is called an arc of the circle.

The arc of a circle is denoted by the symbol " $\frown$ "



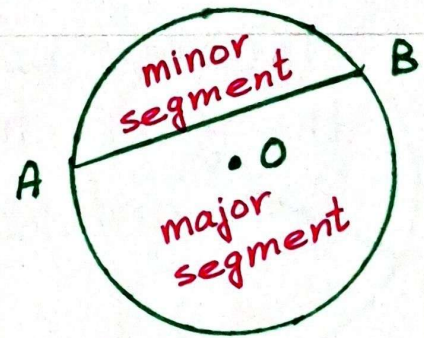
In the adjoining figure,  $\widehat{PQ}$  denotes the arc PQ of the circle with centre O.

Generally, a chord divides the circumference of a circle into two unequal parts. The greater one is known as the major arc and the shorter one is known as the minor arc.  $\widehat{PQ}$  is minor arc,  $\widehat{PRQ}$  is major arc.



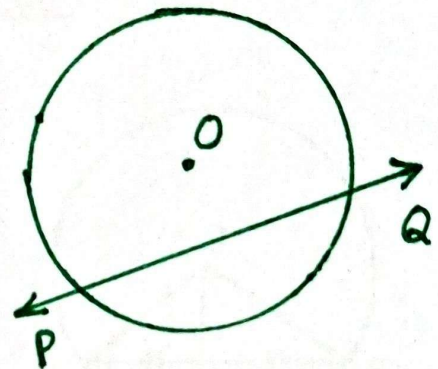
Class 9<sup>th</sup>, MathematicsSegment

A chord of a circle divides its circular region into two parts. Each part of the circular region is called a segment of the circle.

Secant

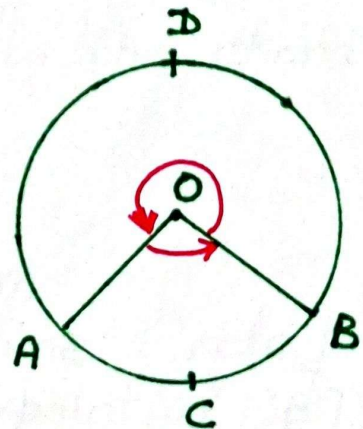
A line which meets a circle in two points is called a secant of the circle.

Here,  $PQ$  is a secant of the circle with centre  $O$ .

Angle at the centre

If two radii are drawn to join the extremities of an arc to the centre of a circle, then the angle formed between those two radii is known as the angle subtended by the arc at the centre of the circle.

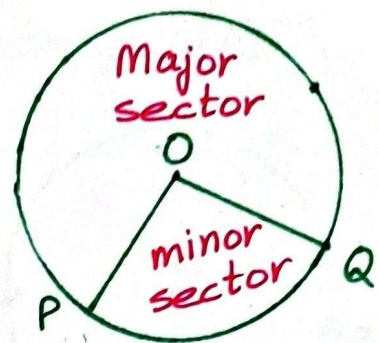
Here,  $\widehat{ACB}$  subtends  $\angle AOB$  at the centre and  $\widehat{BDA}$  subtends reflex  $\angle AOB$  at the centre.

Sector

The part of the plane region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.

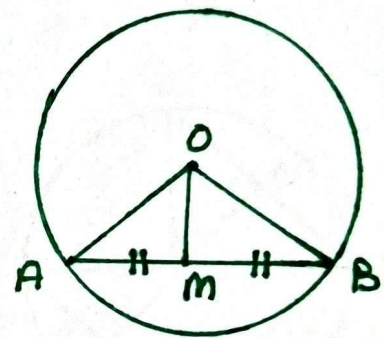


The part containing the minor arc is called minor sector and the part containing the major arc is called major sector.



## CHORD PROPERTIES OF CIRCLES

Theorem 1:- The straight line drawn from the centre of a circle to bisect a chord, which is not a diameter, is perpendicular to the chord.



Given:- A chord AB of a circle with centre O, and OM bisects the chord AB.

To prove:-  $OM \perp AB$

Construction:- Join OA and OB

Proof:- In  $\triangle OAM$  and  $\triangle OBM$

$$OA = OB \quad [\text{radii of same circle}]$$

$$AM = MB \quad [M \text{ is mid-point of } AB]$$

$$OM = OM \quad [\text{Common}]$$

$$\triangle OAM \cong \triangle OBM \quad [\text{S.S.S. rule of congruency}]$$

$$\angle AMO = \angle OMB \quad [\text{C.P.C.T}]$$

$$\angle AMO + \angle OMB = 180^\circ \quad [AMB \text{ is a straight line}]$$

$$\angle AMO = 90^\circ \quad [\because \angle AMO = \angle OMB]$$

Hence  $OM \perp AB$

Theorem 2:- (Converse of theorem 1)

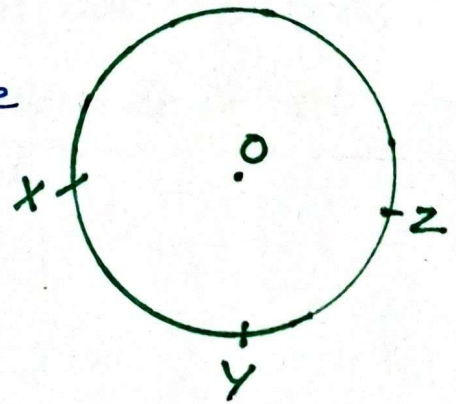
The perpendicular to a chord from the centre of the circle bisects the chord.



Class 9<sup>th</sup>, Mathematics

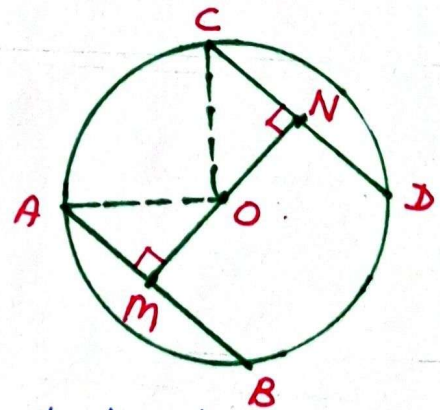
**Theorem 3:-** There is one and only one circle which passes through three given points not in a straight line.

Here,  $x, y$  and  $z$  are the three non collinear points, that is they are not in a straight line.



**Theorem 4:-** Equal chords of a circle are equidistant from the centre.

**Given:-**  $AB$  and  $CD$  are chords of a circle with centre  $O$ , and  $AB = CD$



**To Prove:-**  $AB$  and  $CD$  are equidistant from  $O$  that is if  $OM \perp AB$  and  $ON \perp CD$ , then  $OM = ON$

**Construction:-** Join  $OA$  and  $OC$

**Proof:-** We know that perpendicular from centre bisects the chord

$$\Rightarrow AM = \frac{1}{2} AB \quad \text{and} \quad CN = \frac{1}{2} CD$$

$$\Rightarrow AM = CN \quad [AB = CD \text{ (given)}]$$

Now, In  $\triangle OAM$  and  $\triangle OCN$

$$AM = CN \quad [\text{proved above}]$$

$$\angle AMO = \angle CNO \quad [\text{each} = 90^\circ, OM \perp AB \text{ and } ON \perp CD]$$

$$OA = OC \quad [\text{Radii of same circle}]$$

$$\therefore \triangle OAM \cong \triangle OCN \quad [\text{R.H.S. rule of congruency}]$$

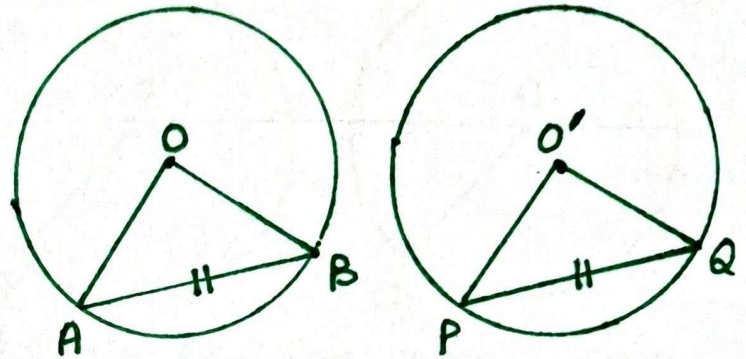
$$OM = ON \quad [\text{c.p.c.t.}]$$



Class 9<sup>th</sup>, MathematicsCHORDS AND ARCS PROPERTIES

**Theorem 7:-** In equal circles (or in the same circle), equal chords cut off equal arcs.

**Given:-** AB and PQ are chords of two equal circles with centres O and O' and  $AB = PQ$



**To Prove:-** Arc AB = Arc PQ

**Construction:-** Join OA, OB, O'P and O'Q

**Proof:-** In  $\triangle OAB$  and  $\triangle O'PQ$

$$OA = O'P \quad [\text{Radii of equal circles}]$$

$$OB = O'Q$$

$$AB = PQ \quad [\text{Given}]$$

$$\therefore \triangle OAB \cong \triangle O'PQ \quad [\text{S.S.S. rule of congruency}]$$

$$\angle AOB = \angle PO'Q \quad [\text{C.p.c.t.}]$$

$$\widehat{AB} = \widehat{PQ}$$

**Conversely,** In equal circles (or in same circle), if two arcs are equal then their chords are equal.

that is if  $\text{arc } \widehat{AB} = \text{arc } \widehat{PQ}$  then  $AB = PQ$

**Equal arcs (congruent arcs)**

Two arcs of the same circle (or congruent circles) having equal measures are called equal arcs (congruent arcs)



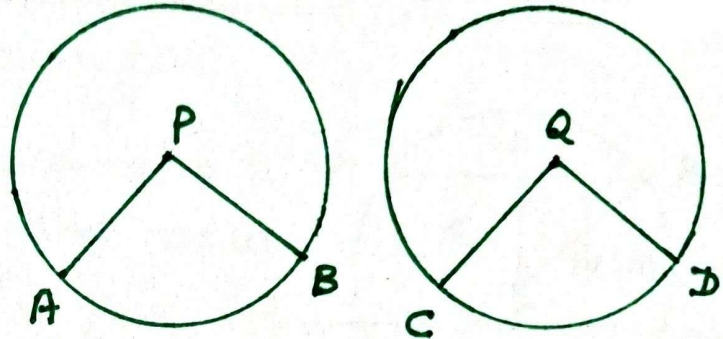
## Class 9, Mathematics

Theorem 6:- In equal circles (or in the same circle), if two arcs subtend equal angles at the centre, they are equal.

that is

If  $\angle APB = \angle CQD$

then  $\widehat{AB} = \widehat{CD}$



Conversely,

If two arcs of equal circles (or in the same circle) are equal, they subtend equal angles at the centre.

So, in same circle (or of equal circles)

- 1) Equal chords subtend equal angles at the centre of the circle.
- 2) Equal angles at the centres make equal chords.

