Tender Heart High School,Sector 33B,Chd.

Class 9, Mathematics

Date: 29.04.2024

Ms. Reena

Chapter-6 Indices / Exponents

$$a = x^{m}$$

$m \rightarrow$ Exponent of power

$x \rightarrow$ Base of power

'a' is mth power of x

x^m is exponential form of 'a'

Indices :-

For any real number 'a' and positive integer m, we define :- $a \times a \times a \ldots$ to m factors $= a^m$

OR $x \times x \times x \ldots$ to m factors $= x^m$

→ The plural of index is indices

We define, $a^0 = 1$ and $a^{-n} = \dfrac{1}{a^n}$

Example:- (i) $3^0 = 1$, $\left(\dfrac{2}{5}\right)^0 = 1$

(ii) $2^{-5} = \dfrac{1}{2^5} = \dfrac{1}{32}$

Fractional Indices

The nth root is denoted by $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$

If $\sqrt[n]{a} = x \Rightarrow a^{\frac{1}{n}} = x \Rightarrow a = x^n$

Example:- $\sqrt{9} = 9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}} = 3$

$\sqrt[3]{8} = 8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2$

Note :- $(-9)^{\frac{1}{2}}, (-16)^{\frac{1}{4}}, (-64)^{\frac{1}{6}}$ are not defined

For any rational number $\dfrac{p}{q}$, we define

$$a^{\frac{p}{q}} = \left(a^{\frac{1}{q}}\right)^p = (a^p)^{\frac{1}{q}} = \sqrt[q]{a^p}$$

Ms. Reena

Laws of Indices

For all real numbers 'a' and 'b' and all rational numbers 'm' and 'n', we have :-

(i) $a^m \times a^n = a^{m+n}$

(ii) $\dfrac{a^m}{a^n} = a^{m-n}, \ a \neq 0$

(iii) $(a^m)^n = a^{mn}$

(iv) $(ab)^n = a^n b^n$

(v) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, \ b \neq 0$

(vi) $a^{-n} = \dfrac{1}{a^n}, \ a \neq 0$

(vii) $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

(viii) $a^m = a^n \Rightarrow m = n$, where $a > 0, a \neq 1$

(ix) $a^m = b^m, \ m \neq 0 \Rightarrow a = b$, where 'a' and 'b' are positive

(x) $p^m \times q^n \times r = p^a q^b r^c$

$\Rightarrow m = a, \ n = b, \ s = c$

where p, q, r are different primes

Example :- Simplify $\dfrac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 2^2 \times 5^n}$

$$\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 2^2 \times 5^n} = \frac{5^2 \times 5^{n+1} - 6 \times 5^{n+1}}{9 \times 5^n - 4 \times 5^n}$$

$$= \frac{25 \times 5^{n+1} - 6 \times 5^{n+1}}{9 \times 5^n - 4 \times 5^n}$$

$$= \frac{5^{n+1} \times (25-6)}{5^n \times (9-4)} = \frac{19 \times 5^{n+1}}{5 \times 5^n} = \frac{19 \times 5^{n+1}}{5^{n+1}} = 19$$

Ms. Reena

Example 2:- Solve

(i) $\sqrt{\left(\frac{3}{5}\right)^{1-2x}} = 4\left(\frac{17}{27}\right)$

$\Rightarrow \left[\left(\frac{3}{5}\right)^{1-2x}\right]^{\frac{1}{2}} = \frac{125}{27} \Rightarrow \left(\frac{3}{5}\right)^{\frac{1-2x}{2}} = \frac{125}{27}$

$\Rightarrow \left(\frac{3}{5}\right)^{\frac{1}{2}-x} = \left(\frac{5}{3}\right)^{3} \Rightarrow \left(\frac{3}{5}\right)^{\frac{1}{2}-x} = \left(\frac{3}{5}\right)^{-3}$

Since base is same, so we compare powers

$\Rightarrow \frac{1}{2} - x = -3 \Rightarrow x = \frac{1}{2} + 3 = 3.5$

(ii) $\sqrt{\left(5^{\circ} + \frac{2}{3}\right)} = (0.6)^{2-3x}$

$\Rightarrow \sqrt{\left(1 + \frac{2}{3}\right)} = \left(\frac{6}{10}\right)^{2-3x}$

$\Rightarrow \left(\frac{5}{3}\right)^{\frac{1}{2}} = \left(\frac{3}{5}\right)^{2-3x} \Rightarrow \left(\frac{5}{3}\right)^{\frac{1}{2}} = \left(\frac{5}{3}\right)^{-2+3x}$

$\Rightarrow \frac{1}{2} = -2 + 3x \Rightarrow 3x = \frac{1}{2} + 2 \Rightarrow 3x = \frac{5}{2}$

$\Rightarrow x = \frac{5}{6}$ Hence, $x = \frac{5}{6}$ is the solution of the given equation.