Tender Heart High School,Sector 33B,Chd.

Class 9, Mathematics

Date 22.04.2024

Chapter- 1 Rational and Irrational Numbers.-II

Teacher: Ms. Reena Tyagi

NUMBERS

- Real(R)
  - Irrational Numbers e.g. $\sqrt{2}, \sqrt{3}, \sqrt{19}, \ldots$
  - Rational Numbers(Q) e.g. $-3, 0, \frac{7}{2}, \frac{-3}{5}, \ldots$
    - Integers (I or Z) $\ldots -2, -1, 0, 1, 2, \ldots$
      - Negative Integers $\ldots, -3, -2, -1$
      - Whole Numbers (W) $0, 1, 2, 3, \ldots$
        - Zero
        - Natural Numbers (N) $1, 2, 3, \ldots$
    - Non-Integers $\frac{5}{14}, \frac{7}{2}, -\frac{3}{5}, \ldots$
- Imaginary (roots of negative numbers)

Real Numbers:- Collection of all rational numbers together with all irrational numbers form the collection of real numbers.

Rational Numbers:- Any number that can be expressed as $\frac{p}{q}$ where p and q are both integers, $q \neq 0$ and p and q have no common factors except 1 i.e. p and q are co-prime.

Ms. Reena

Irrational Numbers:– A number that cannot be expressed in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$, p and q have no common factors (except 1) is called an irrational number.

e.g. $\sqrt{2}, \sqrt{3}, \sqrt{7}, -\sqrt{6}, \sqrt{20}, \frac{1}{\sqrt{5}}, 2+\sqrt{3}$ etc.

Note:– Rational number can be terminating or non terminating. e.g. $3, -\frac{7}{2}, 0.666\text{---} = 0.\overline{6}$ etc

Irrational number is a non-terminating and non-repeating decimal.

e.g. $\sqrt{2}, \sqrt{3}, 0.10110111011110\ldots, \sqrt[3]{2}, \sqrt[4]{9}$, etc

Example 1:– Check whether the following are rational or irrational numbers.

(i) $\frac{1}{2} = 0.5$ (terminating) $\longrightarrow$ Rational Number

(ii) $\sqrt{3} = 1.7320\ldots$ (non terminating non-repeating)
So, it is irrational number.

(iii) $\frac{2}{3} = 0.666\ldots = 0.\overline{6}$ (non terminating and repeating)
So, it is rational number.

(iv) $0.20220222 0\ldots$ (non terminating non-repeating)
So, it is irrational number.

(v) $\frac{22}{7} = 3.142857\ldots$

Here, $\pi = \frac{22}{7}$ is a rational number as it is defined as a ratio of 2 integers.
and $\pi = 3.14158\ldots$ is an irrational number as it is non terminating and non repeating

Class 9, Mathematics

Chapter-1 (Part-II) Ms. Reena

Example 2:- (i) Insert four irrational numbers between $3\sqrt{2}$ and $2\sqrt{3}$

Solution:- Consider the squares of $3\sqrt{2}$ and $2\sqrt{3}$

$(3\sqrt{2})^2 = 9 \times 2 = 18$ and $(2\sqrt{3})^2 = 4 \times 3 = 12$

As $18 > 17 > 15 > 14 > 13 > 12$, it follows that

$\sqrt{18} > \sqrt{17} > \sqrt{15} > \sqrt{14} > \sqrt{13} > \sqrt{12}$

$\Rightarrow 3\sqrt{2} > \sqrt{17} > \sqrt{15} > \sqrt{14} > \sqrt{13} > 2\sqrt{3}$

(ii) Find two irrational number between $\sqrt{2}$ and $\sqrt{7}$

Solution:- Consider the square of $\sqrt{2}$ and $\sqrt{7}$

$\Rightarrow (\sqrt{2})^2 = 2$ and $(\sqrt{7})^2 = 7$

As $2 < 3 < 5 < 7$, it follows that

$\sqrt{2} < \sqrt{3} < \sqrt{5} < \sqrt{7}$

$\Rightarrow$ Hence, two irrational numbers are $\sqrt{3}$ and $\sqrt{5}$

Note:- Since infinitely many irrational number lie between two distinct irrational numbers. Therefore, we can consider other numbers also.

SURDS:-

Let 'a' be a rational number and 'n' be a positive integer such that $\sqrt[n]{a}$ is irrational. Then $\sqrt[n]{a}$ is called a surd or a radical of order n.

Examples:-

1) $\sqrt{3}$, $\sqrt[3]{4}$, $\sqrt[4]{6}$ and $\sqrt[7]{10}$ are surds of order 2, 3, 4 and 7 respectively.
2) $\sqrt[3]{27}$ is not a surd, since $\sqrt[3]{27} = 3$ which is rational.
3) $\sqrt{\pi}$ is not a surd, because π is irrational.
4) $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$ is a surd
5) $\sqrt[3]{4} \times \sqrt[3]{54} = \sqrt[3]{4 \times 54} = \sqrt[3]{216} = \sqrt[3]{6^3} = 6$ is not a surd.

Class 9, Mathematics

Chapter- 1 (Part-II)

Ms. Reena

Example 3:- Simplify $\sqrt{27} - 3\sqrt{75} + 5\sqrt{48} + 2\sqrt{108}$

Solution:- $\sqrt{9\times3} - 3\sqrt{25\times3} + 5\sqrt{16\times3} + 2\sqrt{36\times3}$

$= 3\sqrt{3} - 3\times5\sqrt{3} + 5\times4\sqrt{3} + 2\times6\sqrt{3}$

$= 3\sqrt{3} - 15\sqrt{3} + 20\sqrt{3} + 12\sqrt{3}$

$= (3-15+20+12)\sqrt{3}$

$= 20\sqrt{3}$

Example 4:- Write in ascending order

$4\sqrt{5}, 5\sqrt{3}, 10, 3\sqrt{7}, 6\sqrt{2}$

Solution:- $4\sqrt{5} = \sqrt{16}\times\sqrt{5} = \sqrt{80}$

$5\sqrt{3} = \sqrt{25}\times\sqrt{3} - \sqrt{75}$

$10 = \sqrt{100}$

$3\sqrt{7} = \sqrt{9}\times\sqrt{7} = \sqrt{63}$

$6\sqrt{2} = \sqrt{36}\times\sqrt{2} = \sqrt{72}$

Now, $63 < 72 < 75 < 80 < 100$

$\Rightarrow \sqrt{63} < \sqrt{72} < \sqrt{75} < \sqrt{80} < \sqrt{100}$

$\Rightarrow 3\sqrt{7} < 6\sqrt{2} < 5\sqrt{3} < 4\sqrt{5} < 10$

Example 5:- Arrange the following numbers in ascending order $\sqrt{3}, \sqrt[3]{5}, \sqrt[4]{8}$

Solution:- L.C.M. of roots i.e. 2, 3 and 4 is 12

$\Rightarrow$

$\sqrt{3} = 3^{\frac{1}{2}} = (3^6)^{\frac{1}{12}} = (729)^{\frac{1}{12}}$

$\sqrt[3]{5} = 5^{\frac{1}{3}} = (5^4)^{\frac{1}{12}} = (625)^{\frac{1}{12}}$

$\sqrt[4]{8} = 8^{\frac{1}{4}} = (8^3)^{\frac{1}{12}} = (512)^{\frac{1}{12}}$

As $512 < 625 < 729$

$\Rightarrow (512)^{\frac{1}{12}} < (625)^{\frac{1}{12}} < (729)^{\frac{1}{12}}$

$\Rightarrow \sqrt[4]{8} < \sqrt[3]{5} < \sqrt{3}$

Rationalisation of Surds

The process of multiplying a surd by another surd to get a rational number is called rationalisation. Each surd is called rationalising factor of the other surd.

e.g. 1) $(4+\sqrt{3})(4-\sqrt{3}) = (4)^2 - (\sqrt{3})^2 = 16-3 = 13$

2) $\dfrac{1}{\sqrt{5}} = \dfrac{1}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

3) $\dfrac{2}{5-\sqrt{3}} = \dfrac{2}{5-\sqrt{3}} \times \dfrac{5+\sqrt{3}}{5+\sqrt{3}} = \dfrac{2(5+\sqrt{3})}{(5)^2-(\sqrt{3})^2}$

$= \dfrac{2(5+\sqrt{3})}{25-3} = \dfrac{2(5+\sqrt{3})}{22} = \dfrac{5+\sqrt{3}}{11}$

Rule :- Multiply and divide the numerator and denominator of the given expression by the rationalising factor of its denominator and simplify.

Example 1:- Rationalise the denominator of the following

(i) $\dfrac{2}{3+\sqrt{5}} = \dfrac{2}{3-\sqrt{5}} \times \dfrac{3+\sqrt{5}}{3+\sqrt{5}} = \dfrac{2(3+\sqrt{5})}{(3)^2-(\sqrt{5})^2}$

$= \dfrac{2(3+\sqrt{5})}{9-5} = \dfrac{2(3+\sqrt{5})}{4} = \dfrac{3+\sqrt{5}}{2}$

(ii) $\dfrac{17}{3\sqrt{2}+1} = \dfrac{17}{3\sqrt{2}-1} \times \dfrac{3\sqrt{2}+1}{3\sqrt{2}+1} = \dfrac{17(3\sqrt{2}+1)}{(3\sqrt{2})^2-(1)^2}$

$= \dfrac{17(3\sqrt{2}+1)}{18-1} = \dfrac{17(3\sqrt{2}+1)}{17} = 3\sqrt{2}+1$

(iii) $\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \times \dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \dfrac{(\sqrt{2}+\sqrt{3})^2}{(\sqrt{2})^2-(\sqrt{3})^2}$

$= \dfrac{(\sqrt{2})^2+(\sqrt{3})^2+2\sqrt{2}\sqrt{3}}{2-3} = \dfrac{2+3+2\sqrt{6}}{-1}$

$= \dfrac{5+2\sqrt{6}}{-1} = -(5+2\sqrt{6})$

Example 2 If a and b are rational numbers and $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a-b\sqrt{3}$, find the values of 'a' and 'b'.

Solution:- $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{5(7-4\sqrt{3})+2\sqrt{3}(7-4\sqrt{3})}{(7)^2-(4\sqrt{3})^2}$

$= \frac{35-20\sqrt{3}+14\sqrt{3}-8\times 3}{49-48} = \frac{11-6\sqrt{3}}{1}$

Now, $a-b\sqrt{3} = 11-6\sqrt{3}$

On comparing, we get $a = 11$ and $b = 6$

Example 3 (i) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

$= \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} \times \frac{\sqrt{48}-\sqrt{18}}{\sqrt{48}-\sqrt{18}} = \frac{4\sqrt{3}(\sqrt{48}-\sqrt{18})+5\sqrt{2}(\sqrt{48}-\sqrt{18})}{(\sqrt{48})^2-(\sqrt{18})^2}$

$= \frac{4\sqrt{3\times 48}-4\sqrt{3\times 18}+5\sqrt{2\times 48}-5\sqrt{2\times 18}}{48-18}$

$= \frac{4\sqrt{3\times 3\times 4\times 4}-4\sqrt{3\times 3\times 3\times 2}+5\sqrt{2\times 3\times 4\times 4}-5\sqrt{2\times 2\times 3\times 3}}{30}$

$= \frac{4\times 3\times 4-4\times 3\sqrt{6}+5\times 4\sqrt{6}-5\times 2\times 3}{30}$

$= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{30} = \frac{18+8\sqrt{6}}{30} = \frac{9+4\sqrt{6}}{15}$

(ii) $\frac{2}{\sqrt{5}+\sqrt{3}+2} = \frac{2}{(\sqrt{5}+\sqrt{3})+2} \times \frac{(\sqrt{5}+\sqrt{3})-2}{(\sqrt{5}+\sqrt{3})-2}$

$= \frac{2(\sqrt{5}+\sqrt{3}-2)}{(\sqrt{5}+\sqrt{3})^2-(2)^2} = \frac{2(\sqrt{5}+\sqrt{3}-2)}{5+3+2\sqrt{5}\sqrt{3}-4}$

$= \frac{2(\sqrt{5}+\sqrt{3}-2)}{8+2\sqrt{15}-4} = \frac{2(\sqrt{5}+\sqrt{3}-2)}{4+2\sqrt{15}} = \frac{\sqrt{5}+\sqrt{3}-2}{2+\sqrt{15}}$

Again rationlising the denominator, we get

$\frac{\sqrt{5}+\sqrt{3}-2}{2+\sqrt{15}} \times \frac{2-\sqrt{15}}{2-\sqrt{15}}$

contd...

$$= \frac{\sqrt{5}(2-\sqrt{15}) + \sqrt{3}(2-\sqrt{15}) - 2(2-\sqrt{15})}{(2)^2 - (\sqrt{15})^2}$$

$$= \frac{2\sqrt{5} - \sqrt{5\times15} + 2\sqrt{3} - \sqrt{3\times15} - 4 + 2\sqrt{15}}{4-15}$$

$$= \frac{2\sqrt{5} - 5\sqrt{3} + 2\sqrt{3} - 3\sqrt{5} - 4 + 2\sqrt{15}}{-11}$$

$$= \frac{-\sqrt{5} - 3\sqrt{3} - 4 + 2\sqrt{15}}{-11} = \frac{\sqrt{5} + 3\sqrt{3} + 4 - 2\sqrt{15}}{11}$$

Note : In the above example, we can consider $\frac{2}{\sqrt{5}+\sqrt{3}+2}$ as $\frac{2}{\sqrt{5}+(\sqrt{3}+2)}$ also.
i.e. in denominator we can take any two numbers in pair for rationalis

Example 4:- If $x = 2-\sqrt{3}$, find the value of $\left(x - \frac{1}{x}\right)^3$

Solution:- Given $x = 2-\sqrt{3}$

Therefore, $\frac{1}{x} = \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$$= \frac{2+\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

Now, $x - \frac{1}{x} = (2-\sqrt{3}) - (2+\sqrt{3})$

$$= 2-\sqrt{3}-2-\sqrt{3} = -2\sqrt{3}$$

Therefore, $\left(x-\frac{1}{x}\right)^3 = (-2\sqrt{3})^3$

$$= (-2)^3(\sqrt{3})^3$$

$$= -8 \times 3\sqrt{3} = -24\sqrt{3}$$

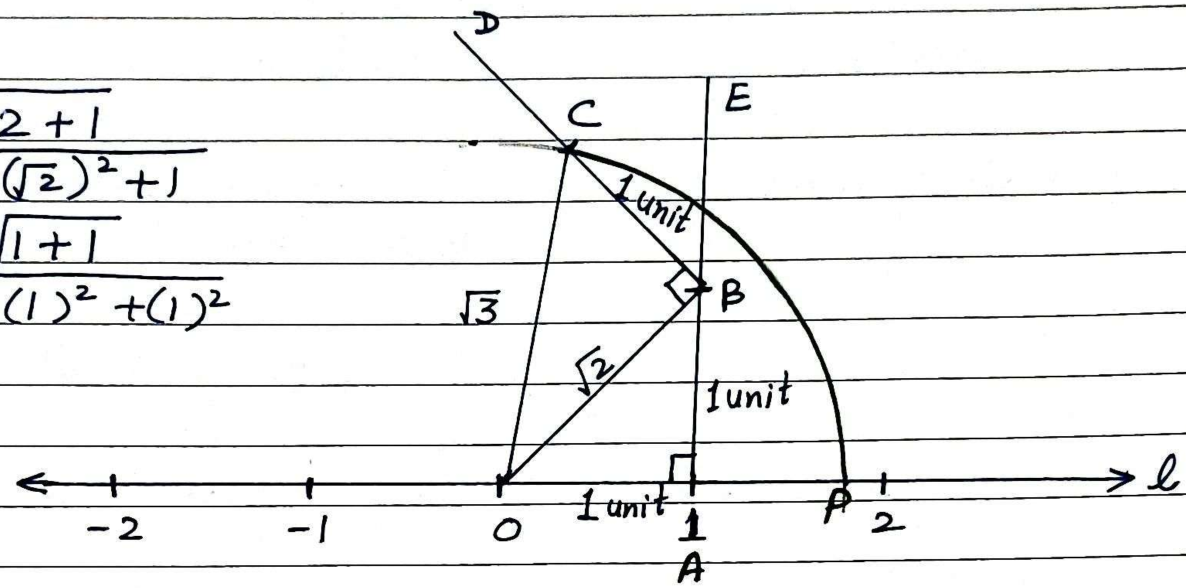
Ms. Reena

Representation of irrational number on number line.

1) $\sqrt{3}$

$\sqrt{3} = \sqrt{2+1}$
$= \sqrt{(\sqrt{2})^2 + 1}$

$\sqrt{2} = \sqrt{1+1}$
$= \sqrt{(1)^2 + (1)^2}$



Steps:-

1) Draw a line and mark numbers on it taking any appropriate scale.
2) At point A make an angle of 90° (using compass or protector)
3) Take $OA = AB = 1$ unit
4) Join OB which is equal to $\sqrt{2}$
5) At point B make an angle of 90° and take $BC = 1$ unit.
6) Join OC which is equal to $\sqrt{3}$
7) With O as centre and radius $= OC$, we draw an arc of a circle to meet the number line 'l' at the point P
8) As $OP = OC = \sqrt{3}$ units, the point P will represent the number $\sqrt{3}$ on the number line.

We know that $\sqrt{1} < \sqrt{2} < \sqrt{3} < \sqrt{4}$

i.e. $\therefore 1 < \sqrt{2} < \sqrt{3} < 2$

So, $\sqrt{3}$ is lying between 1 and 2.

Class 9, Mathematics
Chapter -1 Ms. Reena

To prove $\sqrt{2}$ is an irrational number.

If possible, let $\sqrt{2}$ be rational
Let its simplest form be $\sqrt{2} = \frac{p}{q}$, where
p and q are integers having no common factor, other than 1 and $q \neq 0$

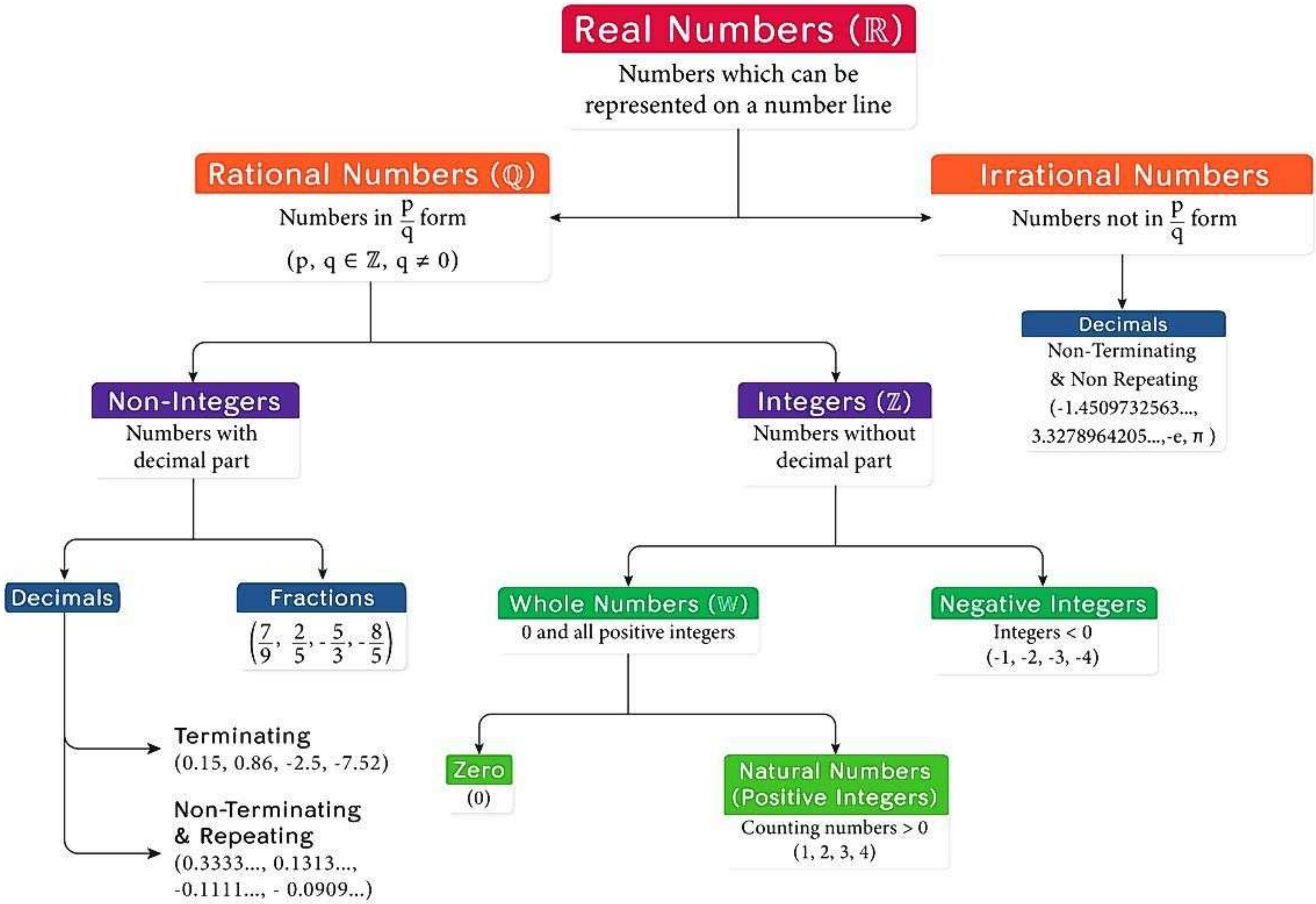
Then, $\sqrt{2} = \frac{p}{q} \Rightarrow p^2 = 2q^2$ ---- (i)

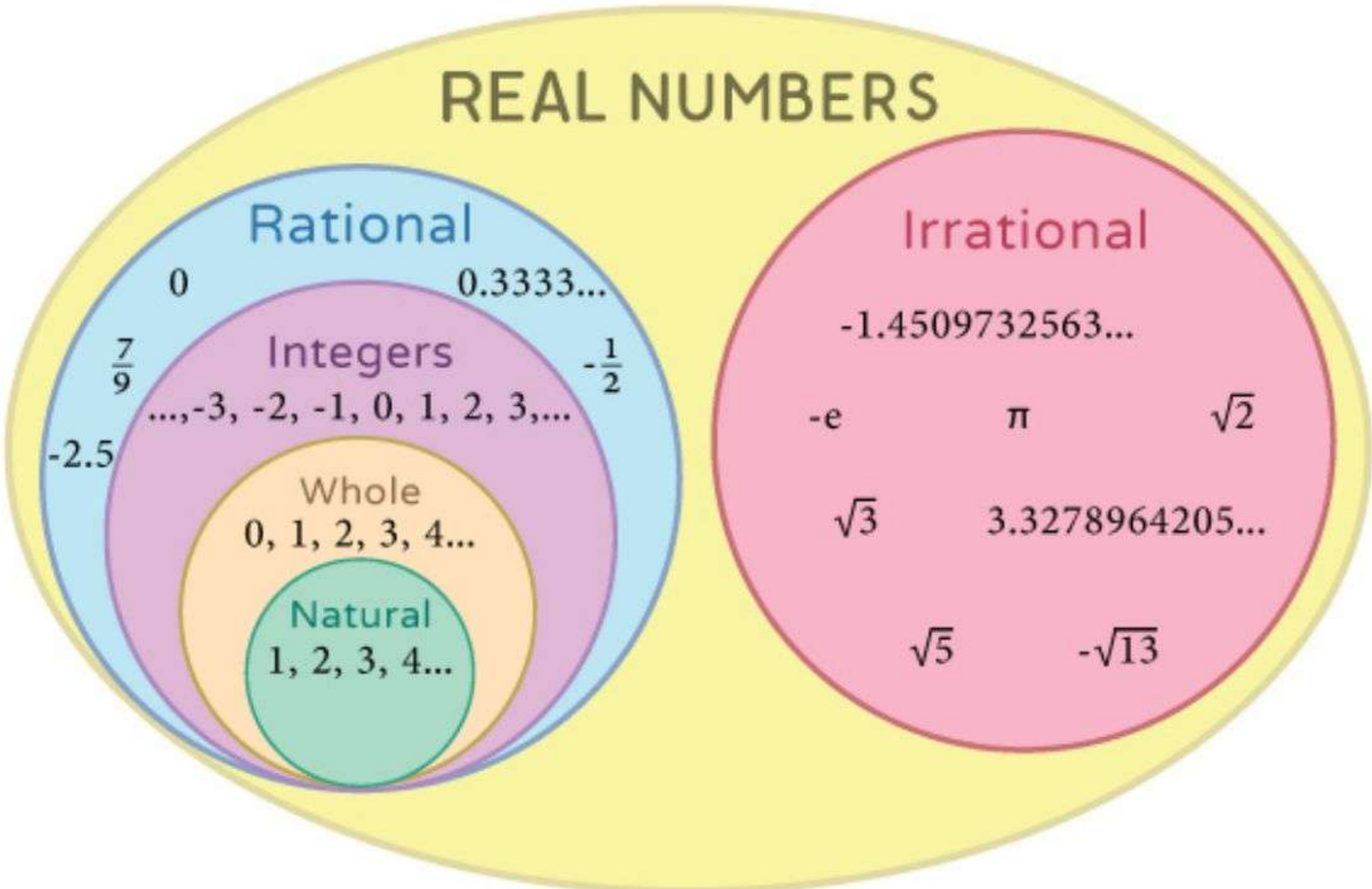
$\Rightarrow p^2$ is even $\Rightarrow p$ is even OR [multiple of 2]

Let $p = 2m$ for some integer m.
Then $p = 2m \Rightarrow p^2 = 4m^2$
$\Rightarrow 2q^2 = 4m^2$ [Using (i)]
$\Rightarrow q^2 = 2m^2 \Rightarrow q^2$ is even
$\Rightarrow q$ is even
Thus, p is even and q is even
This shows that 2 is a common factor of p and q
This contradicts the hypothesis that p and q have no common factor, other than 1
Therefore, $\sqrt{2}$ is not a rational and hence it is irrational.

Note : $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ we can prove in same manner.

(Last page)


Real Numbers (ℝ)
Numbers which can be represented on a number line
Rational Numbers (ℚ)
Numbers in $\frac{p}{q}$ form
($p, q \in \mathbb{Z}, q \neq 0$)
Irrational Numbers
Numbers not in $\frac{p}{q}$ form
Decimals
Non-Terminating & Non Repeating
(-1.4509732563..., 3.3278964205...,-e, π)
Non-Integers
Numbers with decimal part
Integers (ℤ)
Numbers without decimal part
Decimals
Fractions
($\frac{7}{9}, \frac{2}{5}, -\frac{5}{3}, -\frac{8}{5}$)
Whole Numbers (𝕎)
0 and all positive integers
Negative Integers
Integers < 0
(-1, -2, -3, -4)
Terminating
(0.15, 0.86, -2.5, -7.52)
Non-Terminating & Repeating
(0.3333..., 0.1313..., -0.1111..., - 0.0909...)
Zero
(0)
Natural Numbers (Positive Integers)
Counting numbers > 0
(1, 2, 3, 4)


REAL NUMBERS
Rational
0
0.3333...
$\frac{7}{9}$
$-\frac{1}{2}$
-2.5
Integers
...,-3, -2, -1, 0, 1, 2, 3,...
Whole
0, 1, 2, 3, 4...
Natural
1, 2, 3, 4...
Irrational
-1.4509732563...
-e
π
$\sqrt{2}$
$\sqrt{3}$
3.3278964205...
$\sqrt{5}$
$-\sqrt{13}$