Tender Heart High School, Sector 33B, Chd.

Class 9, Mathematics

Chapter 1 Rational and Irrational Numbers

Date 15.04.2024

[Mrs. Reena]

Introduction:-

* **Natural Numbers**: The counting numbers are called natural numbers i.e. N = 1, 2, 3, 4, ...
* **Whole Numbers**: All counting numbers together with 0 form the collection of all whole numbers
  W = 0, 1, 2, 3, 4, 5, ...
* **Integers**: All whole numbers and the negatives of all counting numbers form the collection of all integers.
  I or Z = ... −4, −3, −2, −1, 0, 1, 2, 3, 4, ...
* **Fractions**: The numbers of the form $\frac{a}{b}$, where 'a' and 'b' are natural numbers.
  e.g. $\frac{2}{3}, \frac{4}{7}, \frac{9}{61}$ ...

Terminating Decimals:-

In order to express a fraction $\frac{p}{q}$ in decimal form, we divide p by q. If after a finite number of steps, no remainder is left, then we call it a terminating decimal

e.g. $\frac{1}{4} = 0.25$, $\frac{3}{8} = 0.375$, $\frac{14}{5} = 2.8$

Note:- A fraction $\frac{p}{q}$ is a terminating decimal only when prime factors of denominator are out of 2 and 5 only.

e.g. $20 = 2 \times 2 \times 5$, terminating

$30 = 2 \times \underline{3} \times 5$ non-terminating

$252 = 2 \times 2 \times \underline{3} \times \underline{3} \times \underline{7}$ non-terminating

$60 = 2 \times 2 \times \underline{3} \times 5$ non-terminating

$50 = 2 \times 5 \times 5$ terminating

Ms. Reena

Repeating (or Recurring) Decimals / non-terminating

→ A decimal in which a digit or a set of digits repeats continually, is called a repeating or a recurring or a periodic or a circulating decimal.

e.g. $\frac{1}{3} = 0.3333\ldots = 0.\dot{3}$ or $0.\overline{3}$

$\frac{2}{3} = 0.666\ldots = 0.\dot{6}$ or $0.\overline{6}$

$\frac{11}{6} = 1.8333\ldots = 1.8\dot{3}$ or $1.8\overline{3}$

Example 1 Express the following decimals as a rational number in simplest form $\frac{p}{q}$ or vulgar fraction.

(i) $0.\overline{6}$

Let $x = 0.\overline{6} \Rightarrow x = 0.666\ldots$ (i)

Multiply both side by 10, we get

$10x = 6.666\ldots$ (ii)

On subtracting (i) from (ii), we get

$9x = 6 \Rightarrow x = \frac{6}{9} = \frac{2}{3}$

Hence, $0.\overline{6} = \frac{2}{3}$ i.e. vulgar fraction.

(ii) $5.\overline{36}$

Let $x = 5.\overline{36} \Rightarrow x = 5.363636\ldots$ (i)

Multiply both side by 100 (since two digits are repeating) we get $100x = 536.363636\ldots$ (ii)

On subtracting (i) from (ii), we get

$99x = 531$

$\Rightarrow x = \frac{531}{99} = \frac{59}{11}$

Hence, $5.\overline{36} = \frac{59}{11}$

(iii) $0.4\overline{56}$

Let $x = 0.4565656 \ldots$ (i)

First we multiply by 10 to shift 4

i.e. $10x = 4.565656 \ldots$ (ii)

Since, two digits are repeating so, we multiply equation (ii) by 100, we get

$1000x = 456.5656 \ldots$ (iii)

On subtracting equation (ii) from (iii), we get

$990x = 452 \Rightarrow x = \frac{452}{990} = \frac{226}{495}$

Hence, $0.4\overline{56} = \frac{226}{495}$

Note :- We subtract the two equation only when digits after decimals ar exactly same. So, we multiply by 10, 100, 1000...

Rational Numbers

Any number that can be expressed in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$

i.e. $p, q \in I$, $q \neq 0$ and p, q are coprime
[coprime → no common factor except 1]

Note :-

(i) Every rational number is expressible either as a terminating decimal or as a repeating decimal.

(ii) Every terminating decimal is a rational number

(iii) Every repeating decimal is a rational number.

(iv) The sum, difference and product of two rational numbers is always rational

(v) When a rational number is divided by any non-zero rational, we always get a rational.

Class 9, Mathematics
Chapter – 1

Ms. Reena

There are infinitely many rational numbers between two rational numbers.
i.e. If x and y are two rational numbers then $x < \frac{1}{2}(x+y) < y$

By repeated use of above property, we can find infinitely many rational numbers.

Example 1:- Find a rational number between $\frac{3}{5}$ and $\frac{5}{7}$

Solution:- Using $x < \frac{1}{2}(x+y) < y$

$$\Rightarrow \frac{3}{5} < \frac{1}{2}\left(\frac{3}{5} + \frac{5}{7}\right) < \frac{5}{7}$$

$$\Rightarrow \frac{3}{5} < \frac{1}{2}\left(\frac{21+25}{35}\right) < \frac{5}{7} \Rightarrow \frac{3}{5} < \frac{1}{2} \times \frac{46}{35} < \frac{5}{7}$$

$$\Rightarrow \frac{3}{5} < \frac{23}{35} < \frac{5}{7}$$

2nd Method

L.C.M of denominator 5 and 7 is 35

$$\Rightarrow \frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35} \text{ and } \frac{5}{7} = \frac{5 \times 5}{7 \times 5} = \frac{25}{35}$$

So, new fractions are $\frac{21}{35}$ and $\frac{25}{35}$

The fractions lying between $\frac{21}{35}$ and $\frac{25}{35}$

$$\frac{21}{35} < \frac{22}{35} < \frac{23}{35} < \frac{24}{35} < \frac{25}{35}$$

Here, we can take any fraction as answer.

Note :- We can use any method for finding rational numbers. Here, 2nd method is more convenient.

Example 2:- Find ten rational numbers between $-\frac{2}{3}$ and $\frac{1}{4}$

Solution:- L.C.M of 3 and 4 is 12

$\Rightarrow -\frac{2}{3} = \frac{-2\times4}{3\times4} = \frac{-8}{12}$ and $\frac{1}{4} = \frac{1\times3}{4\times3} = \frac{3}{12}$

Now, rational numbers between $\frac{-8}{12}$ and $\frac{3}{12}$ are

$$\frac{-8}{12} < \frac{-7}{12} < \frac{-6}{12} < \frac{-5}{12} < \frac{-4}{12} < \frac{-3}{12} < \frac{-2}{12} < \frac{-1}{12} < 0 < \frac{1}{12} < \frac{2}{12} < \frac{3}{12}$$

Example 3:- Find four rational numbers between $\frac{1}{4}$ and $\frac{1}{3}$

Solution:- L.C.M of 4 and 3 is 12

$\Rightarrow \frac{1}{4} = \frac{1\times3}{4\times3} = \frac{3}{12}$ and $\frac{1}{3} = \frac{1\times4}{3\times4} = \frac{4}{12}$

Here, the rational numbers are $\frac{3}{12}$ and $\frac{4}{12}$

In order to find four rational number we multiply each by five (i.e. one extra)

$\Rightarrow \frac{3\times5}{12\times5} = \frac{15}{60}$ and $\frac{4\times5}{12\times5} = \frac{20}{60}$

$$\Rightarrow \frac{15}{60} < \frac{16}{60} < \frac{17}{60} < \frac{18}{60} < \frac{19}{60} < \frac{20}{60}$$

$$\Rightarrow \frac{1}{4} < \frac{16}{60} < \frac{17}{60} < \frac{18}{60} < \frac{19}{60} < \frac{1}{3}$$

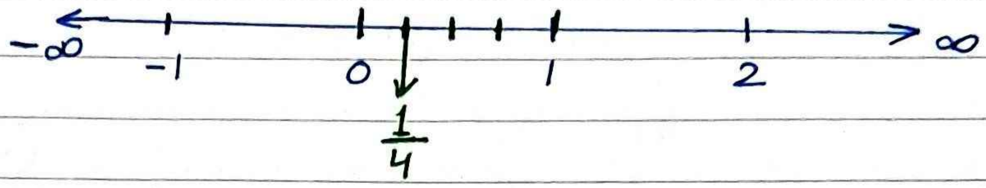
Note:- In the above example we can multiply by any number greater than 4.

Class 9, Mathematics
Chapter - 1

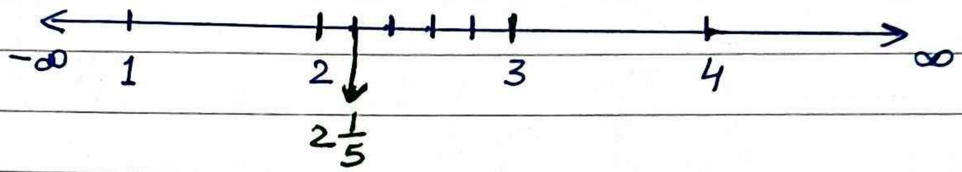
Ms. Reena

Representation of rational number on number line

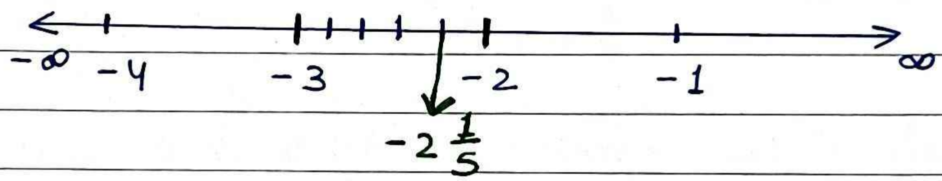
1) $\frac{1}{4}$



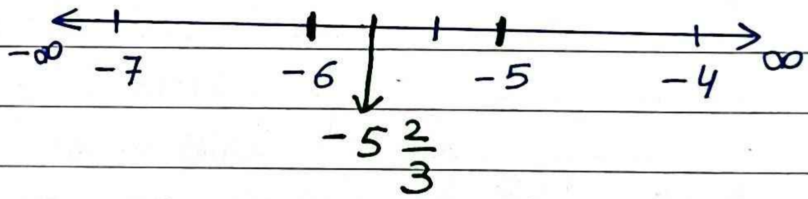
2) $2\frac{1}{5}$



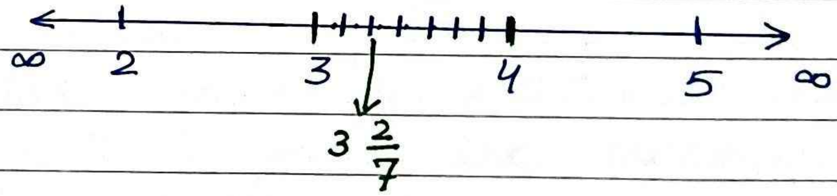
3) $-2\frac{1}{5}$



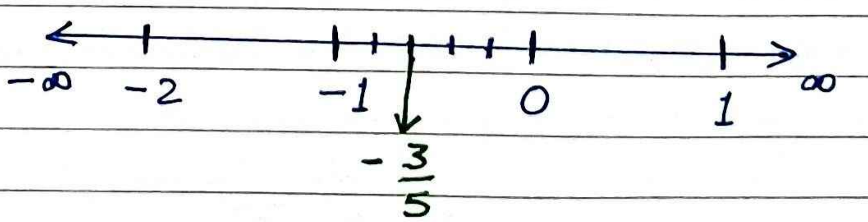
4) $\frac{-17}{3} = -5\frac{2}{3}$



5) $\frac{23}{7} = 3\frac{2}{7}$



6) $-\frac{3}{5}$



last page