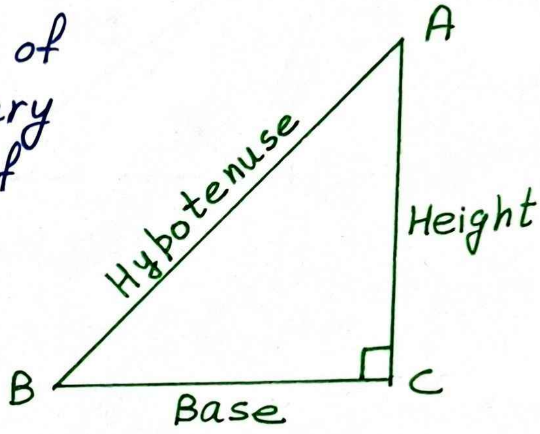
Class 9, Mathematics

Date 5.08.2024

Ms. Reena

Chapter –10

Pythagoras Theorem



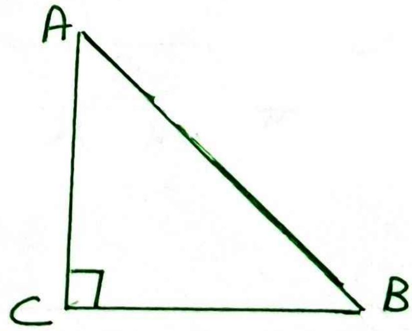
Pythagoras, a Greek Philospher of sixth century B.C. discovered a very important and useful property of right angled triangles, named Pythagoras property.

In a right angled triangle, the sides have special names. The side opposite to the right angle is called hypotenuse and the other two sides are base and height (altitude)

Theorem :- In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

i.e. $AB^2 = AC^2 + BC^2$

(Note : Proof is not required)



Converse of Pythagoras Theorem

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

i.e. If $AB^2 = AC^2 + BC^2$ then $\angle C = 90°$

→ Pythagoras theorem was earlier given by an ancient mathematician Baudhayan (about 800 B.C.)

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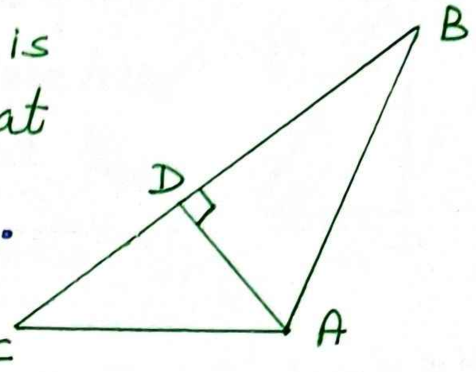
Example 1:- In triangle ABC, AD is perpendicular to BC. Prove that

$AB^2 + CD^2 = AC^2 + BD^2$



Solution:- In $\triangle ABD$, $\angle ADB = 90^\circ$

$\therefore AB^2 = AD^2 + BD^2$ ------ (i)

(By Pythagoras theorem)

In $\triangle ACD$, $\angle ADC = 90^\circ$

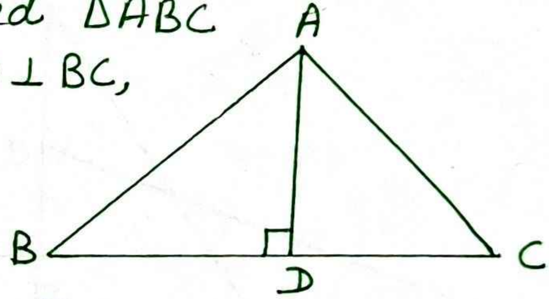
$\therefore AC^2 = AD^2 + CD^2$ ------ (ii)

Subtracting (ii) from (i), we get

$AB^2 - AC^2 = BD^2 - CD^2 \Rightarrow AB^2 + CD^2 = AC^2 + BD^2$

Example 2:- In an acute-angled $\triangle ABC$ in which $\angle B$ is acute and $AD \perp BC$, prove that :-

$AC^2 = AB^2 + BC^2 - 2BC \times BD$



Solution:- In $\triangle ADB$, $\angle ADB = 90^\circ$

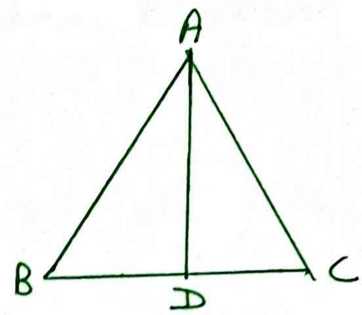
$\therefore AD^2 + BD^2 = AB^2$ ------ (i) [By Pythagoras Theorem]

In $\triangle ADC$, $\angle ADC = 90^\circ$

$$\begin{aligned}\therefore AC^2 &= AD^2 + DC^2 \text{ ---- (ii)}\\ &= AD^2 + (BC - BD)^2\\ &= AD^2 + (BC^2 + BD^2 - 2BC \times BD)\\ &= AD^2 + BD^2 + BC^2 - 2BC \times BD\\ &= AB^2 + BC^2 - 2BC \times BD \quad \text{[using (i)]}\end{aligned}$$

Hence, $AC^2 = AB^2 + BC^2 - 2BC \times BD$

Example 3:- In an equilateral ΔABC, if $AD \perp BC$, prove that, $3AB^2 = 4AD^2$



Solution:- In ΔADB and ΔADC, we have

$AB = AC$, $\angle B = \angle C = 60^\circ$ and

$\angle ADB = \angle ADC = 90^\circ$

$\therefore \Delta ADB \cong \Delta ADC$

so, $BD = DC$

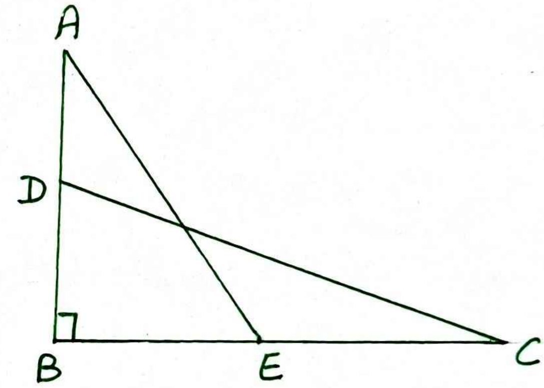
From right ΔADB, we have $AB^2 = AD^2 + BD^2$

$\Rightarrow AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 = AD^2 + \frac{1}{4}BC^2$

$\Rightarrow 4AB^2 = 4AD^2 + BC^2$

$\Rightarrow 4AB^2 = 4AD^2 + AB^2 \Rightarrow 3AB^2 = 4AD^2$

Example 4:- In the given figure, $AE = DC = 13$ cm, $BE = 5$ cm, $\angle ABC = 90^\circ$ and $AD = EC = x$ cm. Calculate the length of AB and the value of x.



Solution:- In ΔABE, $\angle B = 90^\circ$,

$\therefore AE^2 = AB^2 + BE^2 \Rightarrow AB^2 = AE^2 - BE^2$

$\Rightarrow AB^2 = (13)^2 - (5)^2 = 169 - 25 = 144$

$\Rightarrow AB = 12$ cm

From figure, $BD = AB - AD = (12 - x)$ cm

and $BC = BE + EC = (5 + x)$ cm.

In ΔBCD, $\angle B = 90^\circ$

$\therefore CD^2 = BD^2 + BC^2$

$\Rightarrow (13)^2 = (12 - x)^2 + (5 + x)^2$

$\Rightarrow 169 = 144 + x^2 - 24x + 25 + x^2 + 10x$

$\Rightarrow 169 = 169 + 2x^2 - 14x$

$\Rightarrow 2x^2 - 14x = 0 \Rightarrow 2x(x - 7) = 0 \Rightarrow x = 7$ cm