Tender Heart High School, Sector 33B, Chd.

Class 9, Mathematics

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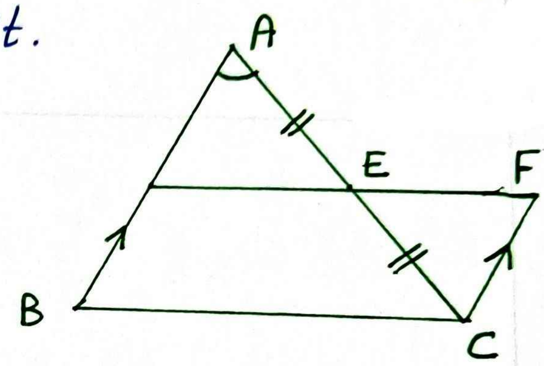
Chapter-9
Mid-point and Equal Intercepts Theorems

Theorem 1 (Mid-Point Theorem)

The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.



Given:- A ΔABC in which D and E are the mid-points of AB and AC respectively.

To prove:- $DE \parallel BC$ and $DE = \frac{1}{2} BC$

Construction:- Through C, draw $CF \parallel BA$, meeting DE produced at F.

Proof:- In ΔAED and ΔCEF, we have

(i) $AE = CE$ (Given, E is the mid-point of AC)

(ii) $\angle AED = \angle CEF$ (Vert. opp. $\angle$'s)

(iii) $\angle EAD = \angle ECF$ (Alt. int. $\angle$'s are equal, as $BA \parallel CF$ and AC is the transverval.)

$\therefore \Delta AED \cong \Delta CEF$ [AAS-axiom]

$\Rightarrow DE = EF$ and $AD = CF$ [c.p.c.t.] ---- (1)

and $AD = BD$ [Given, D is mid-point of AB] -(2)

$\Rightarrow BD = CF$ [From (1) and (2)]

Since $BD = CF$ and $BD \parallel CF$ [by const.]

$\therefore$ DBCF is a parallelogram

$\Rightarrow DE \parallel BC$ and $DF = BC$

$\Rightarrow DE \parallel BC$ and $DE = \frac{1}{2} DF = \frac{1}{2} BC$

Hence, $DE \parallel BC$ and $DE = \frac{1}{2} BC$

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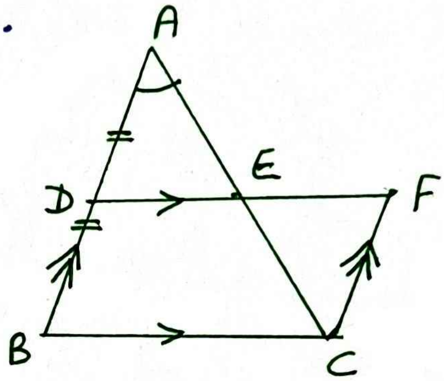
Theorem 2 (Converse of Mid-Point Theorem)
A straight line drawn through the mid-point of one side of a triangle parallel to the other side, bisects the third side.



Given :- A ΔABC in which D is the mid-point of AB and $DE \parallel BC$ meets AC at E.

To prove :- $AE = EC$

Construction :- Draw $CF \parallel BA$, meeting DE produced at F.

Proof :- BCFD is a parallelogram [$DE \parallel BC$ (Given), $CF \parallel BD$ (Const.)]

$\Rightarrow$ $CF = BD$

$CF = AD$ [$\because BD = AD$ (Given)]

In ΔAED and ΔCEF, we have

$AD = CF$ [Proved above]

$\angle AED = \angle CEF$ [vert. opp. $\angle$'s]

$\angle EAD = \angle ECF$ [Alternate interior $\angle$'s]

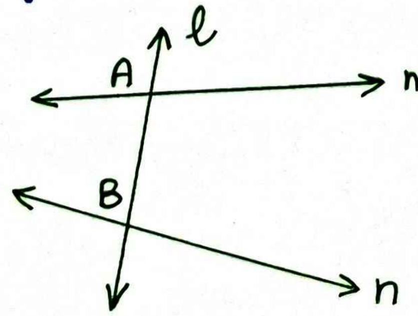
$\Delta AED \cong \Delta CEF$ [AAS-axiom]

$\Rightarrow$ $AE = CE$ [c.p.c.t.]

Hence, E is the mid-point of AC

Intercepts :-

If a line l intersects two straight lines 'm' and 'n' (in the same plane) at points A and B respectively, then the line segment AB is called the intercept on l made by the lines 'm' and 'n'.



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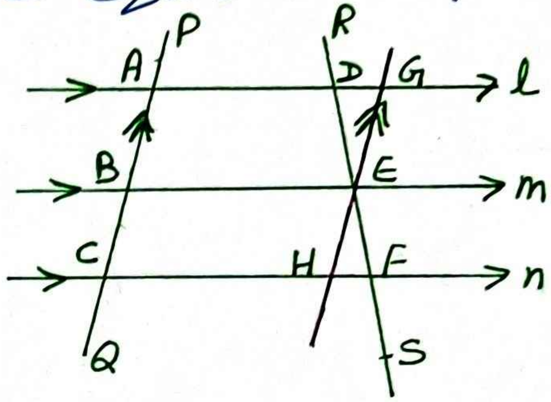
Theorem 3 (Intercept Theorem)

If a transversal makes equal intercepts on three or more parallel lines, then any other line cutting them will also make equal intercepts.



Given:- Three lines l, m, n such that $l \parallel m \parallel n$.
A transversal PQ cuts them at points A, B, C respectively such that AB = BC and an other line RS cuts them at point D, E, F respectively.

To Prove:- DE = EF

Construction:- Through E, draw a line segment $GH \parallel PQ$, meeting l and n at G and H respectively.

Proof:- ABEG is a parallelogram [$AG \parallel BE$ (given), $BA \parallel EG$ (Const.)]

$\Rightarrow$ GE = AB ---(i) [opp. sides of a ||gm are equal]

Also, BCHE is a parallelogram [$BE \parallel CH$ (given), $CB \parallel HE$ (Const.)]

$\Rightarrow$ BC = EH ---(ii) [opp. sides of a ||gm are equal]

Now, from (i) and (ii)

GE = EH [$\because$ AB = BC (given)]

In ΔGED and ΔHEF, we have

GE = EH

$\angle GED = \angle HEF$ [Vert. opp. $\angle$'s]

$\angle DGE = \angle EHF$ [Alt. int. $\angle$'s are equal, as $l \parallel n$ and GH is a transversal]

$\therefore$ $\Delta GED \cong \Delta HEF$ [AAS axiom]

$\Rightarrow$ DE = EF [c.p.c.t.]

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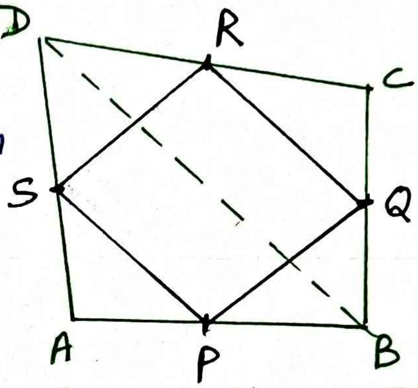
Example 1:- Prove that the figure obtained by joining the mid-points of the adjacent sides of a quadrilateral is a parallelogram.

Given :- P, Q, R, S are the mid-points of the sides AB, BC, CD and DA respectively of quad. ABCD



To Prove :- PQRS is a parallelogram

Construction : Join BD

Proof :- In ΔABD,

$PS \parallel BD$ and $PS = \frac{1}{2} BD$ ---(i)

In ΔBCD, [By mid-point theorem]

$QR \parallel BD$ and $QR = \frac{1}{2} BD$ ---(ii)

Now, from (i) and (ii), we get

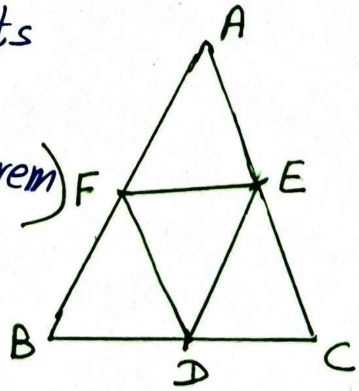
$PS \parallel QR$ and $PS = QR$

$\Rightarrow$ PQRS is a parallelogram.

Example 2:- D, E and F are mid-points of the sides BC, CA and AB respectively of an equilateral triangle ABC. Show that ΔDEF is also an equilateral triangle.

Solution:- As F and E are mid-points of AB and CA respectively of ΔABC.

$FE = \frac{1}{2} BC$ (by mid-point theorem)



Similarly, $FD = \frac{1}{2} AC$

and $DE = \frac{1}{2} AB$

Given, ΔABC is equilateral triangle.

$\therefore \quad AB = BC = CA$

$\Rightarrow \frac{1}{2} AB = \frac{1}{2} BC = \frac{1}{2} CA$

$\Rightarrow \quad DE = FE = FD$

Hence, ΔDEF is also an equilateral triangles.

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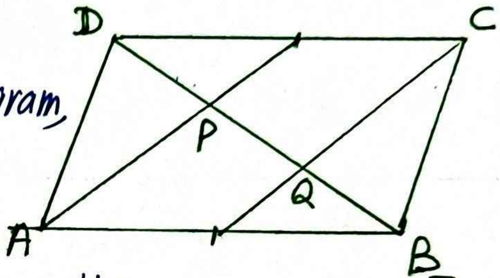
Example 3 In the given figure, ABCD is a parallelogram and E, F are the mid-points of the sides AB, CD respectively. Show that the line segments AF and EC trisect the diagonal BD.



Solution:- As ABCD is a parallelogram,

$AB \parallel DC$

$\Rightarrow AE \parallel FC$

Also $AB = DC$ [opp. sides of a ||gm are equal]

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow AE = FC$ [$\because$ E is mid-point of AB and F is mid-point of CD]

Thus, in the quadrilateral AECF,

$AE = FC$ and $AE \parallel FC$

$\Rightarrow$ AECF is a parallelogram $\Rightarrow AF \parallel EC$

In $\triangle DQC$, $PF \parallel QC$ [$\because AF \parallel EC$ proved above]

and F is mid-point of CD

$\therefore$ P is mid-point of DQ [by converse of mid-point theorem]

$\Rightarrow DP = PQ$ ---- (i)

In $\triangle ABP$, $EQ \parallel AP$ [$\because EC \parallel AF$, from above]

and E is the mid-point of AB

$\therefore$ Q is the mid-point of BP [by converse of mid-point theorem]

$\Rightarrow BQ = PQ$ ---- (ii)

From (i) and (ii), we get $DP = PQ = BQ$

$\Rightarrow$ the line segments AF and EC trisect the diagonal BD.

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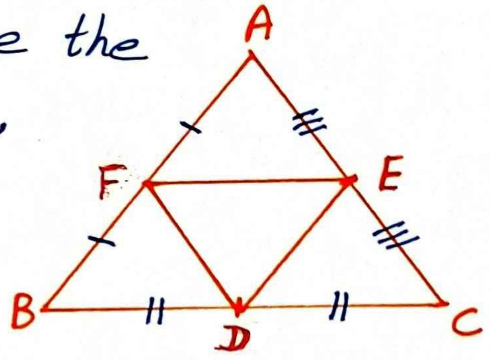
Exercise 9

Solution 1:- Given F, D, E are the mid-points of the sides AB, BC and CA respectively.



By using Mid-point Theorem

i.e. The line segment joining the mid-points of two sides of a triangle is parallel to and half of the third side.

(i) $DE \parallel AB$ and $DE = \frac{1}{2} AB$

Given $AB = 6.2 cm$ $\Rightarrow$ $DE = \frac{1}{2} \times 6.2 = 3.1$ cm

(ii) $DF \parallel AC$ and $DF = \frac{1}{2} AC$

Given $DF = 3.8$ $\Rightarrow$ $AC = 3.8 \times 2 = 7.6$

(iii) Given perimeter of $\Delta ABC = 21 cm$

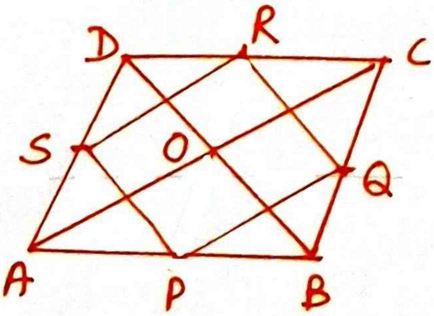
$\Rightarrow$ $AB + BC + AC = 21 cm$

$\Rightarrow$ $6.2 + BC + 7.6 = 21 cm$ $\Rightarrow$ $BC = 21 - 13.8$

$\Rightarrow$ $BC = 7.2 cm$

Now, $EF \parallel BC$ and $EF = \frac{1}{2}(7.2) = 3.6$

Solution 12 Given ABCD is a rhombus and P, Q, R, S are the mid-points of the sides AB, BC, CD and DA respectively.



We need to show that PQRS is a rectangle.

Let the diagonals AC and BD of the rhombus ABCD intersect at O.
In ΔABC, P and Q are the mid-points of side AB and BC respectively,
$\therefore$ By mid-point theorem,

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC$

Similarly, $SR \parallel AC$ and $SR = \frac{1}{2}AC$

$\Rightarrow PQ \parallel SR$ and $PQ = SR$

Thus, in quadrilateral PQRS, $PQ \parallel SR$ and $PQ = SR$ $\Rightarrow$ PQRS is a parallelogram.

In ΔABD, P and S are mid-points of AB and AD respectively. $\therefore PS \parallel BD$

Thus, $EP \parallel OF$ and $PF \parallel EO$

$\therefore$ OEPF is a parallelogram

But $\angle EOF = 90°$ [diagonals of a rhombus intersect at right angles]

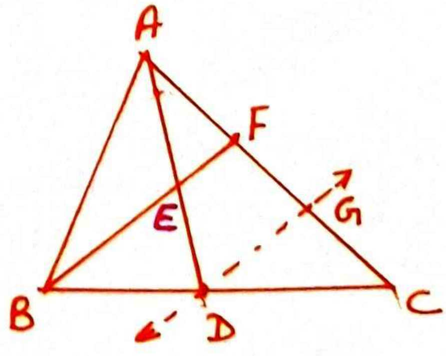
Also, in a parallelogram opposite angles are equal

$\therefore \angle EPF = \angle EOF = 90°$

Thus, PQRS is a parallelogram in which one angle i.e. $\angle P = 90°$

Therefore, PQRS is a rectangle.

Solution 9



Through D, draw a line parallel to BF to meet AC at G.

In ΔAGD, E is the mid-point of side AD and $DG \parallel BF$

i.e. $DG \parallel EF$

$\therefore$ by converse of mid-point theorem, F is the mid point of AG i.e. $AF = FG$ ----- (i)

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In ΔBCF, D is mid-point of side BC and $DG \parallel BF$, therefore, by converse of mid-point theorem, G is mid-point of CF i.e. $FG = GC$ ---- (ii)

From (i) and (ii), we get

$$AF = FG = GC \quad \text{----------} \quad (iii)$$

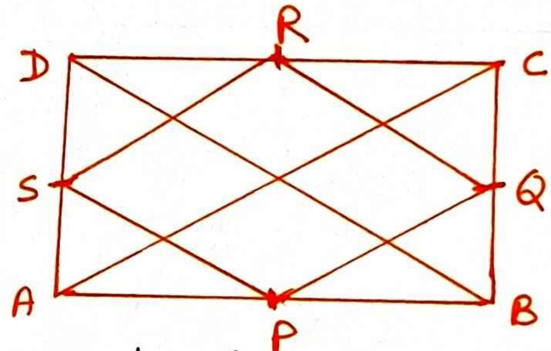
Now, $AC = AF + FG + GC$

$= AF + AF + AF$ [using (iii)]

$= 3\,AF$

$\Rightarrow AF = \frac{1}{3} AC$

Solution 11



Given ABCD is a rectangle and P, Q, R, S are mid-points of the sides AB, BC, CD and DA respectively.

We need to prove that PQRS is a rhombus.

In ΔABC, P and Q are mid-points of the sides AB and BC respectively, therefore, by mid-point theorem, $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$.

Similarly, $SR \parallel AC$ and $SR = \frac{1}{2} AC$

$\Rightarrow PQ \parallel SR$ and $PQ = SR$

Thus, in quadrilateral PQRS, $PQ \parallel SR$ and $PQ = SR$

$\therefore$ PQRS is a parallelogram.

In ΔABD, P and S are mid-points of the sides AB and AD respectively,

$\therefore PS \parallel BD$ and $PS = \frac{1}{2} BD$

But $AC = BD$ [$\because$ diagonals of a rectangle are equal.]

$\therefore PQ = PS$

Thus, PQRS is a rhombus.

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