TENDER HEART HIGH SCHOOL, Sec-33 B, CHD.

Class: 9 Date: 15.7.2024

Subject: Mathematics Teacher: Ms. Reena

Topic: Ch -8 Triangles

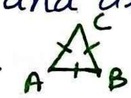
→ A plane figure bounded by three line segments is called a triangle.

→ A line segments forming a triangle are called its sides and each point, where two sides intersect, is called its vertex. (Plural is vertices)

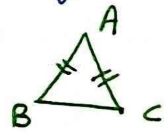
→ A triangle has six elements, namely three sides and three angles.

Types of triangles on the basis of sides

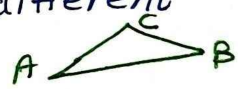
1) Equilateral triangle :- All sides and all angles are equal.



2) Isosceles triangle :- Two sides equal



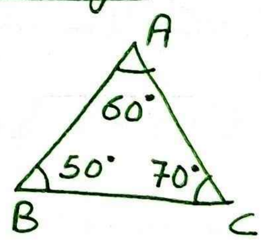
3) Scalene triangle :- All sides are of different length



Types of triangles on the basis of Angles

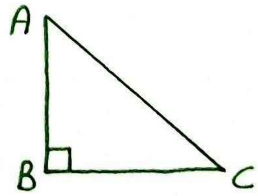
1) Acute-angled triangle :-

A triangle in which every angle measures more than 0° but less than 90°, is called an acute-angled triangle



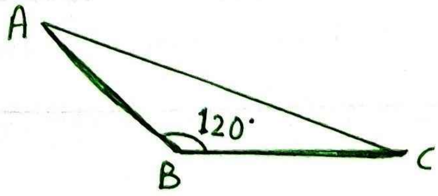
2) Right-angled triangle :-

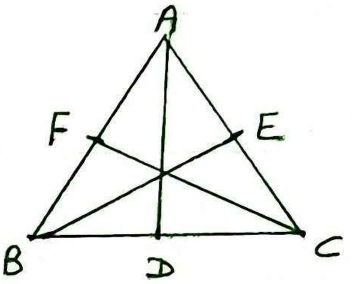
A triangle in which one of the angles measures 90°, is called a right-angled triangle.



3) Obtuse-angled triangle :-

A triangle in which one of the angles measures more than 90° but less than 180°





Medians of a Triangle

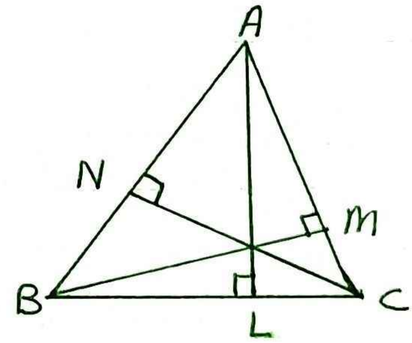
→ It is the line segment joining the mid-point of that side with the opposite vertex.

D, E, F are the mid-points of the sides BC, CA and AB respectively of ΔABC

Thus, AD is the median corresponding to side BC
BE is the median corresponding to side CA
CF is the median corresponding to side AB

→ The point of intersection of the medians of a triangle are concurrent, i.e. they intersect at the same point.

The point of intersection of the medians of a triangle is called its centroid.



Altitudes of a Triangle

→ The altitude of a triangle corresponding to any side is the length of perpendicular from the opposite vertex to that side.

In ΔABC, we have $AL \perp BC$, $BM \perp CA$ and $CN \perp AB$

The point of intersection of the altitude of a triangle is called its orthocentre.

CONGRUENCE OF TRIANGLES

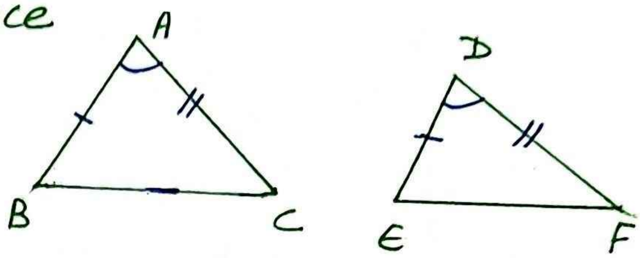
Two geometrical figures, having exactly the same shape and size are known as congruent figures.

For congruence, we use the symbol $\cong$

Thus, two line segments are congruent if they have the same length.

Two angles are congruent if they have the same measures.

Criterion for Congruence



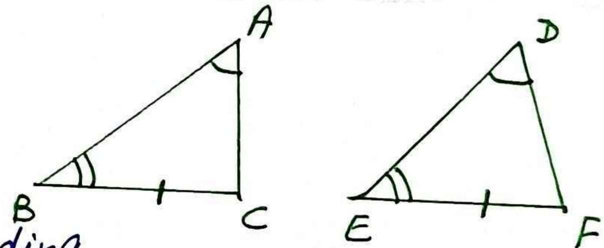
(i) (SAS-axiom)

If two triangles have two sides and the included angle of the one equal to the corresponding sides and the included angle of the other, then the triangles are congruent.

In the given figure, in ΔABC and ΔDEF, we have

$AB = DE$, $AC = DF$ and $\angle A = \angle D$

$\therefore \Delta ABC \cong \Delta DEF$ [By SAS-axiom]



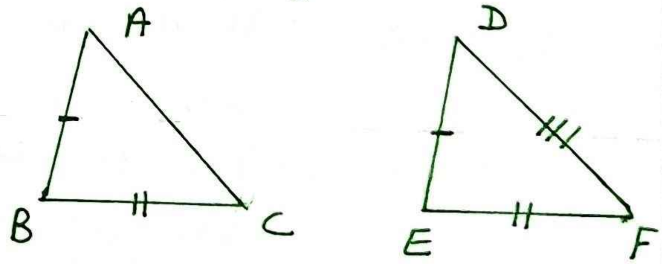
(ii) (AAS-axiom)

If two triangles have two angles and a side of the one equal to the corresponding two angles and the corresponding side of the other, then the triangles are congruent.

In ΔABC and ΔDEF, we have

$\angle A = \angle D$, $\angle B = \angle E$ and $BC = EF$

$\therefore \Delta ABC \cong \Delta DEF$ [By AAS-axiom]



(iii) (SSS-axiom)

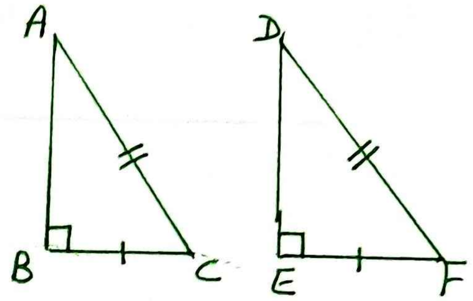
If two triangles have three sides of the one equal to the corresponding three sides of the other, then the triangles are congruent.

In ΔABC and ΔDEF, we have

$AB = DE$, $BC = EF$, $AC = DF$

$\therefore \Delta ABC \cong \Delta DEF$ [By SSS-axiom]

(iv) (RHS - axiom) :-

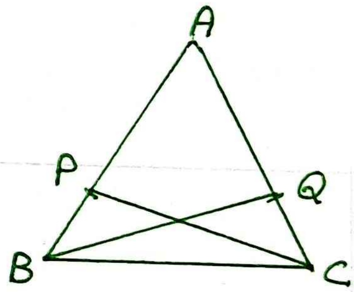


In two right-angled triangles if the hypotenuse and one side of the one are equal to the hypotenuse and the corresponding side of the other, then the triangles are congruent. In ΔABC and ΔDEF, we have

Hyp. AC = Hyp. DF

$BC = EF$ and $\angle ABC = \angle DEF = 90°$

$\therefore \Delta ABC \cong \Delta DEF$ [By RHS axiom]



Example 1 In ΔABC, $AB = AC$. If P is a point on AB and Q is a point on AC such that $AP = AQ$

Prove that (i) $\Delta APC \cong \Delta AQB$

(ii) $\Delta BPC \cong \Delta CQB$

Proof:- (i) In ΔAPC and ΔAQB, we have

$AB = AC$ [Given]

$AP = AQ$ [Given]

$\angle CAP = \angle BAQ$ [Common]

$\therefore \Delta APC \cong \Delta AQB$ [SAS - axiom of congruence]

(ii) Given $AB = AC$ and $AP = AQ$

$\Rightarrow AB - AP = AC - AQ \Rightarrow BP = CQ$

Now, In ΔBPC and ΔCQB, we have

$BP = CQ$ [proved above]

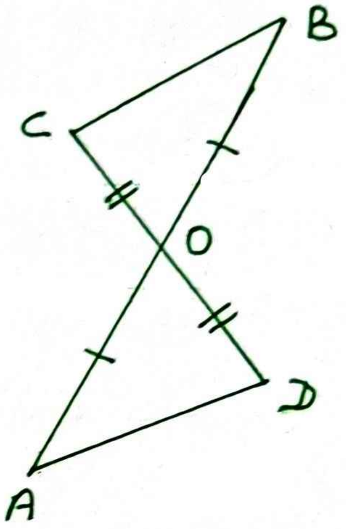
$PC = QB$ [c.p.c.t] as $\Delta APC \cong \Delta AQB$

$BC = CB$ [Common]

$\therefore \Delta BPC \cong \Delta CQB$

c.p.c.t $\longrightarrow$ corresponding parts of congruent triangles are equal

Example 2:- In the adjoining figure,
$OA = OB$ and $OD = OC$



Show that (i) $\triangle AOD \cong \triangle BOC$
(ii) $AD \parallel CB$

Proof:- (i) In $\triangle AOD$ and $\triangle BOC$,

$OA = OB$ (Given)

$OD = OC$ (Given)

$\angle AOD = \angle BOC$ [vert. opp. ∠'s]

$\therefore \triangle AOD \cong \triangle BOC$ (by SAS rule of congruency)

(ii) $\angle OAD = \angle OBC$ (c.p.c.t.)

But these form a pair of alternate angles for line segments AD and BC.

Therefore, $AD \parallel CB$.