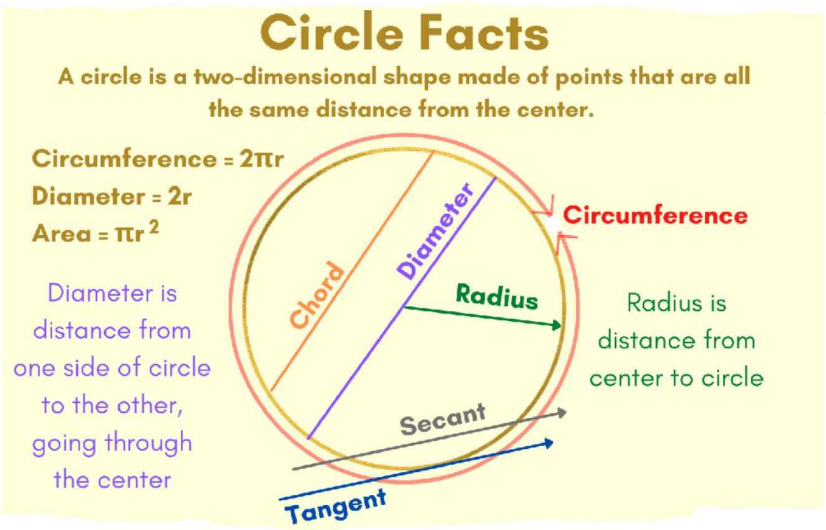
Tender Heart High School ,Sector 33B, Chd.

Chapter 18 Circumference and Area of a Circle

Class 9th | Maths | Date: 25.11.2024

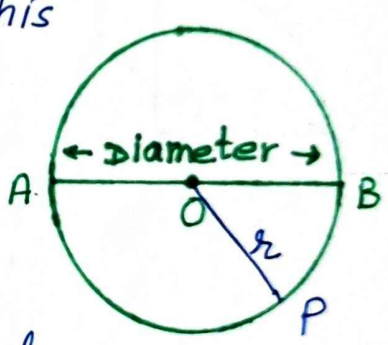


CIRCUMFERENCE AND AREA OF A CIRCLE

The ratio of circumference of any circle to its diameter is constant, and this constant ratio is denoted by π

that is $\frac{\text{circumference}}{\text{diameter}} = \pi$

$\Rightarrow$ circumference $= \pi \times d$



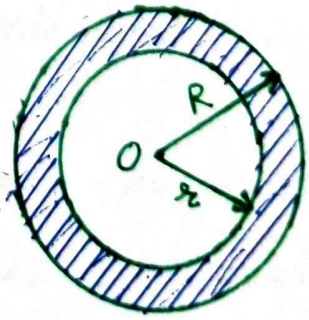
1) Circumference and area of a circle.

If r is the radius of a circle, then

(i) the circumference of the circle $= 2\pi r$

(ii) the area of the circle $= \pi r^2$

2) Area of a circular ring.



If R and r are the radii of the bigger and smaller (concentric) circles, then

Area of ring (shaded portion)

$= \pi (R^2 - r^2)$

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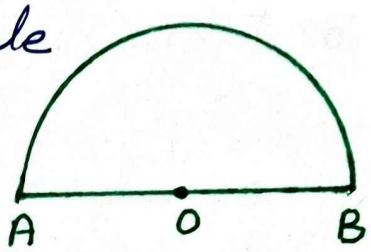
3) **Perimeter and area of a semicircle.**

If 'r' is the radius of a circle, then



(i) the perimeter of the semicircle

$= \frac{1}{2} \times 2\pi r + 2r$

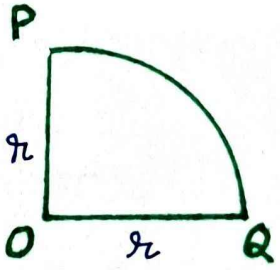
$= (\pi + 2) r$

(ii) the area of the semicircle

$= \frac{1}{2} \pi r^2$

4) **Perimeter and area of a quadrant of a circle.**

If r is the radius of a circle, then



(i) the perimeter of the quadrant $= \frac{1}{4} \times 2\pi r + 2r$

$= \left(\frac{\pi}{2} + 2\right) r$

(ii) the area of the quadrant

$= \frac{1}{4} \pi r^2$

NOTE:- Areas of sectors of circles other than quarter-circle and semicircle are not included.

Students now let us disscuss few examples based on circle, semi-circle and quadrant

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Example 1:- The area of a circle is $301.84\ cm^2$. Calculate :- (i) the radius of the circle in cm (ii) the circumference of the circle, correct to nearest cm.

Solution:- (i) Let the radius of the circle be r cm. Then, its area $= \pi r^2\ cm^2$

Therefore, $\pi r^2 = 301.84 \Rightarrow \frac{22}{7} \times r^2 = 301.84$

$\Rightarrow r^2 = \left(301.84 \times \frac{7}{22}\right) = 96.04 \Rightarrow r = \sqrt{96.04}$

$= 9.8\ cm$

Radius of the circle $= 9.8\ cm$

(ii) Circumference of the circle $= 2\pi r$

$= 2 \times \frac{22}{7} \times 9.8 = 61.6\ cm$

Therefore, circumference of the circle, correct to nearest cm $= 62\ cm$

Example 2:- The area enclosed by the circumferences of two concentric circles is $346.5\ cm^2$. If the circumference of the inner circle is 88 cm, calculate the radius of the outer circle.

Solution:- Let the radius of inner circle be r cm. Then, its circumference $= (2\pi r)$ cm.

$2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = 88 \times \frac{7}{44}$

$\Rightarrow r = 14\ cm$

Radius of the inner circle is, $r = 14\ cm$

Let the radius of the outer circle be R cm.

Then, area of the ring $= (\pi R^2 - \pi r^2)\ cm^2$

$= \pi (R^2 - r^2)\ cm^2 = \frac{22}{7}[R^2 - (14)^2]\ cm^2$

$= \left(\frac{22}{7}R^2 - 616\right) cm^2$

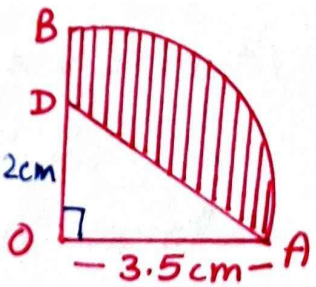
Therefore, $\frac{22}{7}R^2 - 616 = 346.5$

$\Rightarrow \frac{22}{7}R^2 = 962.5 \quad \Rightarrow R^2 = 962.5 \times \frac{7}{22} = 306.25$

$\Rightarrow R = \sqrt{306.25} = 17.5\ cm.$

Hence, the radius of outer circle is 17.5cm

Example 3:- In the figure alongside, OAB is a quadrant of a circle. The radius OA = 3.5cm and OD = 2cm. Calculate the area of the shaded portion.



Solution:- We have

Area of $\Delta OAD = \frac{1}{2} \times OA \times OD = \frac{1}{2} \times 3.5 \times 2 = 3.5 cm^2$

Area of quadrant $OAB = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\ cm^2 = \frac{77}{8}\ cm^2 = 9.625\ cm^2$

Area of shaded portion

= Area of quadrant OAB − Area of ΔOAD

$= (9.625 - 3.5)\ cm^2$

$= 6.125\ cm^2$

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Example 4:- The diameter of the driving wheel of a bus is 140 cm. How many revolutions must the wheel make in order to keep a speed of 66 km/hr?

Solution:- Distance to be covered in 1 min.

$= \left(\frac{66 \times 1000}{60}\right) m = 1100 m$

Radius of the wheel $= \frac{140}{2} = 70_{cm} = 0.70 m$

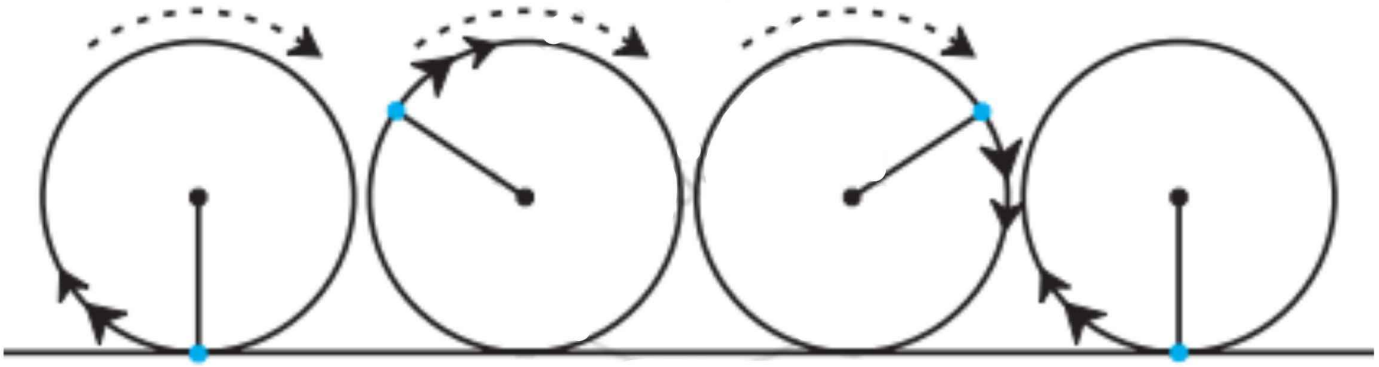
Circumference of the wheel $= 2\pi r$

$= 2 \times \frac{22}{7} \times 0.70 = 4.4 m$

Therefore, number of revolutions per minute

$= \frac{1100}{4.4} = 250$

Hence, the wheel must make 250 revolutions per minute.



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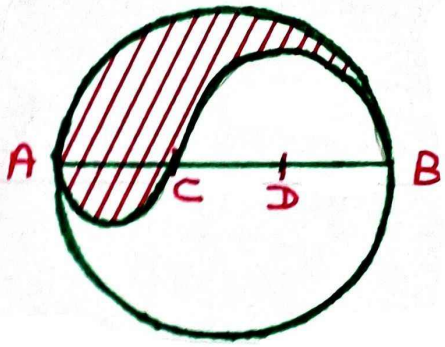
Example 5 Given, radius = 21cm



$\Rightarrow$ diameter $= 2 \times 21 = 42$ cm

So, $AC + CD + DB = 42$ cm

Since $AC = CD = DB$

$\Rightarrow \frac{42}{3} = 14$ cm each

Now, Perimeter of shaded $= \overset{\frown}{AC} + \overset{\frown}{CB} + \overset{\frown}{AB}$

= Circumference of semicircle with radius

$$\left[r_1 = \frac{AC}{2} = \frac{14}{2} = 7\text{cm} \right.$$

$$\left. r_2 = CD = 14\text{cm} \quad \text{and} \quad r_3 = \frac{AB}{2} = 21\text{cm} \right]$$

$$= \pi r_1 + \pi r_2 + \pi r_3$$

$$= \frac{22}{7}(7 + 14 + 21) = \frac{22}{7} \times 42 = 132 \text{ cm}$$

Area of shaded region

$$= \frac{1}{2}\pi r_1^2 + \frac{1}{2}\pi r_2^2 - \frac{1}{2}\pi r_3^2$$

i.e Area of semi-circle $[\overset{\frown}{AC} + \overset{\frown}{AB} - \overset{\frown}{CB}]$

$$= \frac{1}{2} \times \frac{22}{7}\left((7)^2 + (21)^2 - (14)^2\right)$$

$$= \frac{11}{7}(49 + 441 - 196)$$

$$= \frac{11}{7} \times 294 = 462 \text{ cm}^2$$