Tender Heart High School, Sector 33B, Chd.

Class 9, Mathematics
Ch 21 Co-ordinate Geometry

Date : 4.11.2024
Mrs. Reena Tyagi

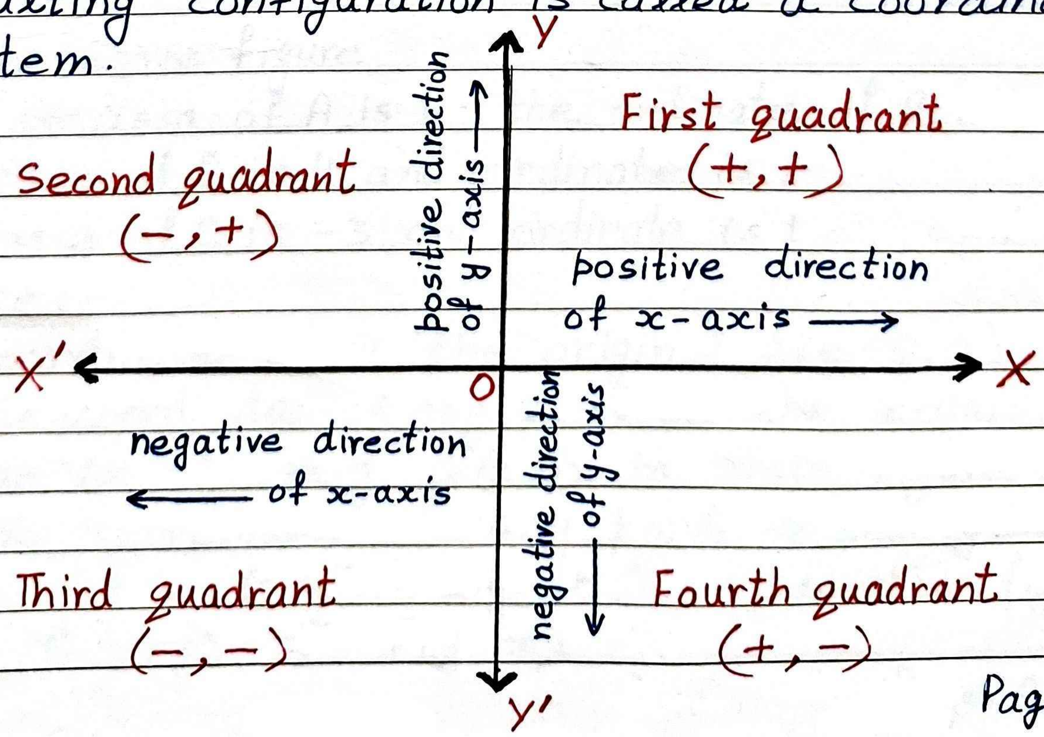
Graphical Solution of Simultaneous Linear Equations

Introduction:-

Coordinate geometry is that branch of Mathematics which deals with the study of geometry by means of algebra. French mathematician René Descartes (1596 - 1650), realised around year 1637 that a line or a curve in a plane can be represented by an algebraic equation. As a result, a new branch of mathematics called Coordinate Geometry came into existence. In honor of his work, the coordinates of a point are often referred to as its Cartesian coordinates, and the coordinate plane as the Cartesian Coordinate Plane.

Coordinate system or Cartesian plane or Cartesian coordinate system

When two numbered lines perpendicular to each other (horizontal and vertical) are placed together so that the two origins coincide then the resulting configuration is called a coordinate system.

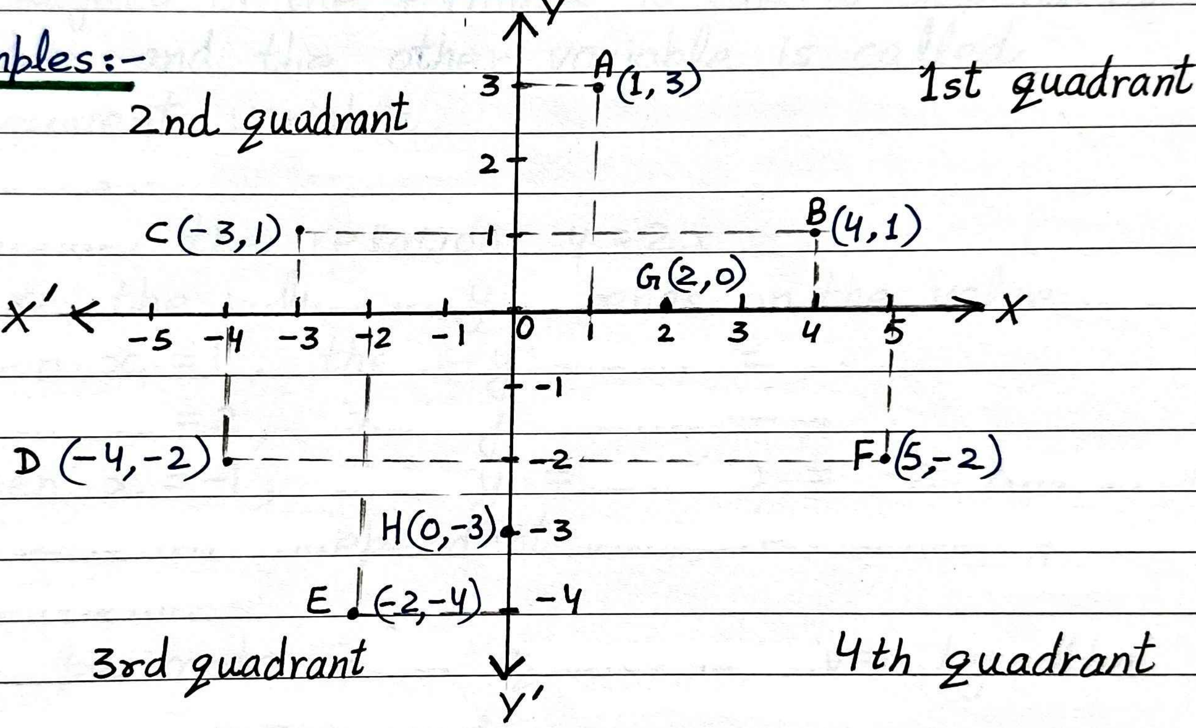


Mrs Reena

The signs of the coordinates of a point in four quadrants can be remembered with the help of the following table:–

Quadrants → Coordinates ↓	1st XOY	2nd YOX'	3rd X'OY'	4th Y'OX
x (abscissa)	+ve	–ve	–ve	+ve
y (ordinate)	+ve	+ve	–ve	–ve

Examples:–



In the above figure,

The abscissa of A is 1; the ordinate of A is 3

Abscissa of B is 4 and ordinate is 1

Abscissa of C is –3 and ordinate is 1 and so on

Note:–

1) Coordinates of O (the origin) are (0, 0)
2) The coordinates of any point on the x-axis would be (x, 0) e.g. G(2, 0) in above figure
3) The coordinates of any point on the y-axis would be (0, y) e.g. H(0, –3) in above figure
4) $P(x, y) \neq P(y, x)$

Mrs Reena

Dependent and Independent Variables

Constant : A symbol having a fixed value is called a constant. e.g. $6, \frac{2}{3}, -7, 0$ etc.

Variable : A symbol which may be assigned different values is called a variable.

There are two types of variables :-
(i) Dependent (ii) Independent.

If there is a formula between two variables, then the subject of the formula is called dependent variable and the other variable is called independent variable.

Examples :-

1) Consider the relation $y = 2x$
Here, the value of y depends on the value of x
When $x = 1$, then $y = 2 \times 1 = 2$
When $x = 2$, then $y = 2 \times 2 = 4$
When $x = -1$, then $y = 2 \times (-1) = -2$ and so on.
So, x is an independent variable, while y is dependent.

2) The perimeter of a square is given by the formula, $P = 4 \times side$
When, side $= 2$, then $P = 4 \times 2 = 8$
When, side $= 5$, then $P = 4 \times 5 = 20$ and so on.
So, Side is an independent variable and P is the dependent variable.

Graph of a Linear Equation

(i) $y = ax$ (ii) $x = a$ (iii) $y = b$
(iv) $y = ax + b$

The graph of a linear equation in two variables is always a straight line.

Mrs. Reena

To draw graphs of linear equations in two variables x and y, we follow the following steps:-

(i) Rewrite the given equation with y as the subject.

(ii) Select any three convenient values of x and find the corresponding values of y for each of the selected value of x.

(iii) Make table of values.

(iv) Draw the axes on the graph and choose suitable scale. (same on both axes).

(v) Plot the points on the graph paper.

(vi) Connect all three points by a straight line.

Example 1:- Draw the graph of $y = -2x + 1$

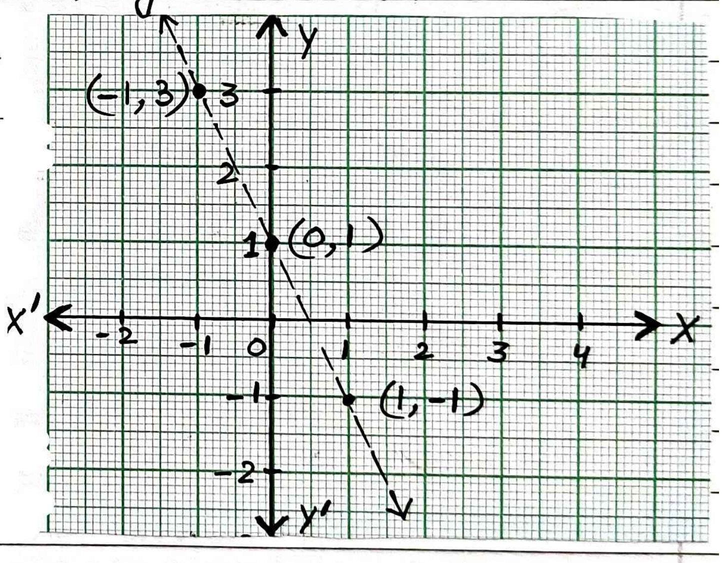
Solution:- $y = -2x + 1$

When $x = 0$, $y = -2 \times 0 + 1 = 1$

When $x = 1$, $y = -2 \times 1 + 1 = -1$

When $x = -1$, $y = -2 \times (-1) + 1 = 3$

x	0	1	-1
y	1	-1	3



Example 2:- Draw the graph of $y = -x$

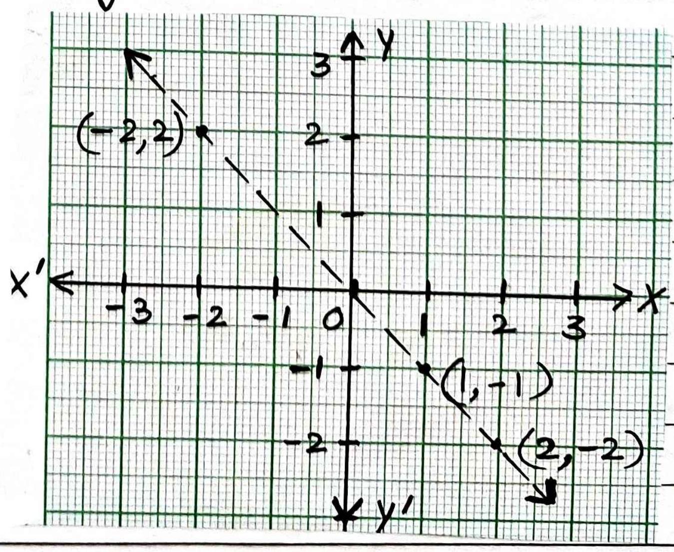
Solution:- $y = -x$

When $x = 1$, $y = -1$

When $x = 2$, $y = -2$

When $x = -2$, $y = 2$

x	1	2	-2
y	-1	-2	2



Graphical Solution of Simultaneous Linear Equation

Teacher : Ms. Reena

System of Simultaneous Linear Equations

→ A set of equations with two or more variables in which the number of equations is the same as the number of variables, is called a system of equations.

→ Two equations whose graphs intersect at a point named by an ordered pair of numbers that satisfies both equations are called simultaneous equations.

→ The coordinates of the point of intersection give the common or simultaneous solution of the two given linear equations.

→ The general form of equations are

$$a_1x + b_1y + c_1 = 0 \quad \text{and} \quad a_2x + b_2y + c_2 = 0$$

If $x = p$ and $y = q$ satisfy the equations then (p, q) is called a solution of the given equations.

Example 1:- Check if $x = 3, y = 4$ is a solution of the linear equation $4x - 3y = 0$ and $2x + 3y - 18 = 0$

Solution :- Substituting $x = 3$ and $y = 4$ in $4x - 3y = 0$

$L.H.S = 4 \times 3 - 3 \times 4 = 12 - 12 = 0$ and $R.H.S = 0$

Thus, $x = 3$ and $y = 4$ satisfy the given equation

Now, substituting $x = 3$ and $y = 4$ in $2x + 3y - 18 = 0$

$L.H.S. = 2 \times 3 + 3 \times 4 - 18 = 6 + 12 - 18 = 0$ and $R.H.S = 0$

Thus, $x = 3$ and $y = 4$ satisfy the given equation

Hence, $(3, 4)$ is a solution of the given system of simultaneous linear equations.

Ms. Reena

Graphical Solution of a pair of Linear Equations

Let $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ be the system of simultaneous linear equations in two variables x and y

We follow the following procedure :-

Step 1:- Construct two separate tables of (both equations) at least three values of x and y

Step 2:- Plot the three points of the first table of values on a graph sheet by choosing suitable scales on the x-axis and y-axis. Join the plotted points to get a straight line.

This straight line represents the graph of the first equation.

Step 3:- Plot the three points of the second table of values on the same graph sheet using the same scales on the two axes. Join the plotted points to get a straight line.

This straight line represents the graph of the second equation.

Step 4:- The coordinates of the point of intersection of the two lines will be the common solution of the given equations. Write the values of x and y.

Example 2:- Solve the following systems of simultaneous linear equation graphically :-

$$2x+3y=12 \quad \text{and} \quad x-y=1$$

Solution:- Given equation is $2x+3y=12$

$$\Rightarrow y=\frac{12-2x}{3}$$

When $x=3$, $y=\frac{12-2\times3}{3}=2$

When $x=0$, $y=\frac{12-2\times0}{3}=4$

When $x=6$, $y=\frac{12-2\times6}{3}=0$

x	0	3	6
y	4	2	0

Ms. Reena

Now, we plot the points (0,4), (3,2) and (6,0) on a graph paper. Join the points and extend it in both the directions.

Now, the second equation is $x - y = 1$

$\Rightarrow x - 1 = y$ or $y = x - 1$

When, $x = -1$, $y = -1-1 = -2$

When, $x = 0$, $y = 0-1 = -1$

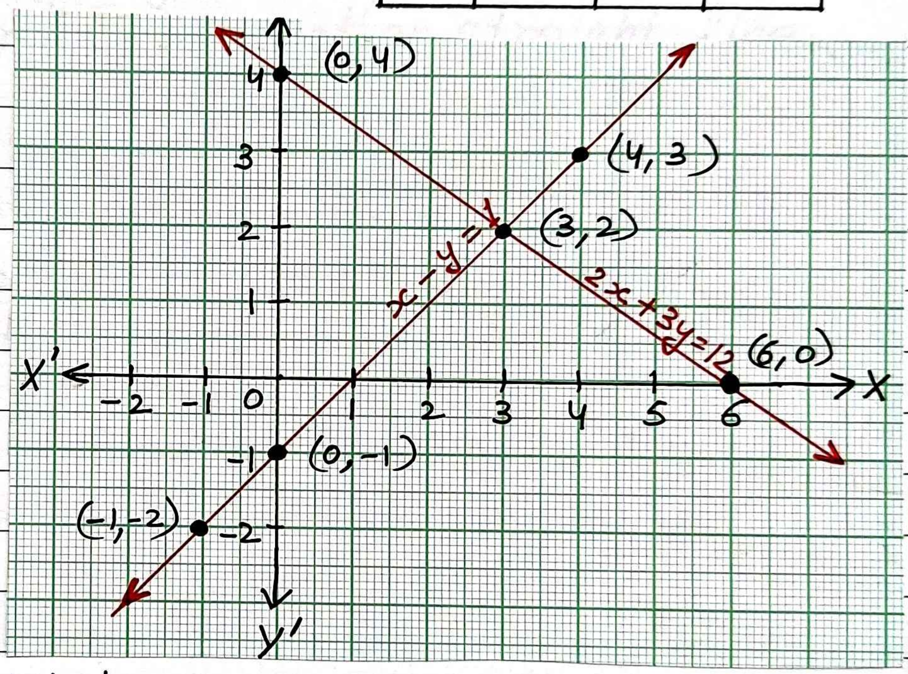
When, $x = 4$, $y = 4-1 = 3$

x	-1	0	4
y	-2	-1	3

On the same graph plot (-1,-2), (0,-1) and (4,3)

The two lines intersect at (3,2)

Therefore, the solution of the given equations is $x = 3$ and $y = 2$



Check :- On substituting $x = 3$ and $y = 2$ in the given equation, we will find that it satisfies both the equations.

Example 3:- Draw the graph of $3x - y - 2 = 0$ and $2x + y - 8 = 0$. Write down the coordinates of the point of intersection and the area of the triangle formed by the lines and the x-axis.

Solution :- Given equation is $3x - y - 2 = 0$

$\Rightarrow y = 3x - 2$

When, $x = 0$, $y = 3(0) - 2 = -2$

When, $x = 1$, $y = 3(1) - 2 = 1$

When, $x = 2$, $y = 3(2) - 2 = 4$

x	0	1	2
y	-2	1	4

The second equation is $2x+y-8=0$

$\Rightarrow y = 8-2x$

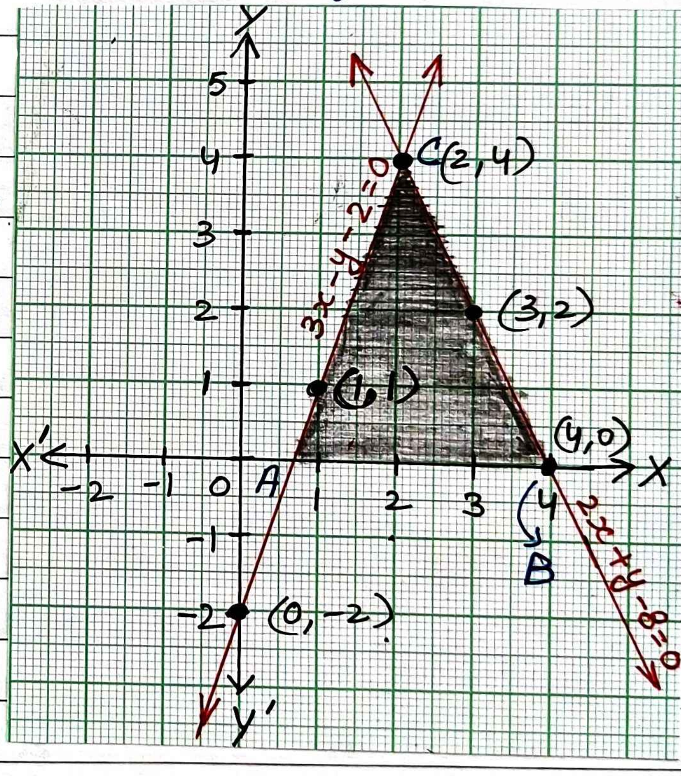
When, $x=2$, $y=8-2(2)=4$

When, $x=3$, $y=8-2(3)=2$

When, $x=4$, $y=8-2(4)=0$

x	2	3	4
y	4	2	0

For equation $3x-y-2=0$, plot points $(0,-2)$, $(1,1)$ and $(2,4)$ on graph and make a straight line. Then, for equation $2x+y-8=0$, plot $(2,4)$, $(3,2)$ and $(4,0)$ on graph and make another straight line.



The straight lines intersect at the point $(2,4)$

Let us find the coordinates of the point of intersection of A of the line $3x-y=2$ by putting $y=0$, we get

$3x-0=2 \Rightarrow x=\frac{2}{3}$

Therefore, $OA=\frac{2}{3}$

$\Rightarrow AB = 4-\frac{2}{3} = \frac{10}{3}$

Area of triangle ABC

$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times \frac{10}{3} \times 4 = \frac{20}{3} = 6\frac{2}{3}$ sq. units.

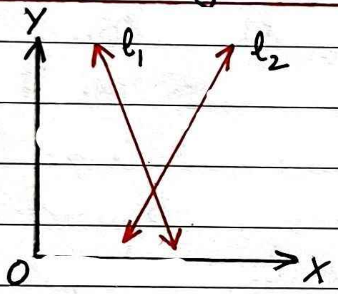
Types of Solutions :–

1) Intersecting lines : Consistent, with unique solution
2) Coincident : Consistent with infinite solution
3) Parallel lines : Inconsistent (No solution)

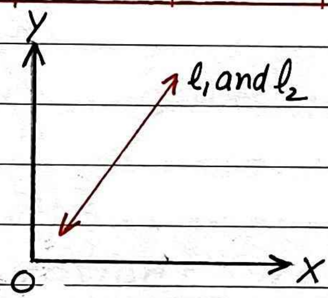
Class 9, Mathematics

Ms. Reena

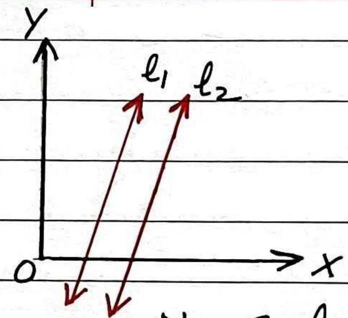
Pair of lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratio
1) $2x+3y+4=0$, $5x+6y+9=0$	$\frac{2}{5}$	$\frac{3}{6}$	$\frac{4}{9}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
2) $x+2y+5=0$, $3x+6y+15=0$	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{5}{15}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
3) $2x-3y+4=0$, $4x-6y+10=0$	$\frac{2}{4}$	$\frac{-3}{-6}$	$\frac{4}{10}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$



Unique solution



Infinitely many solution



No solution

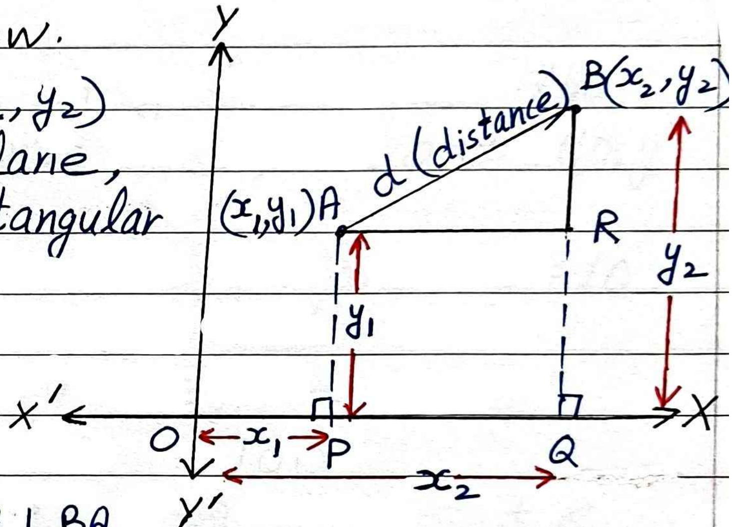
Important Note :- As per ICSE syllabus we pratice questions based on unique solution. Types of solution and its application is not a part of syllabus.

Class 9, Mathematics

Distance Formula

Ms. Reena

→ The coordinates of two points in a plane fix the positions of the points in the plane and also the distance between them. The distance and the coordinates of the two points are related by an algebraic relation which can be deduced as shown below.



→ Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in the plane, OX and OY being the rectangular axes of reference.

Here, AB = distance (d)

$OP = x_1$, $OQ = x_2$

$AP = y_1$, $BQ = y_2$

and $AP \perp OX$, $BQ \perp OX$, $AR \perp BQ$

Now, $AR = OQ - OP = x_2 - x_1$

$BR = BQ - AP$ or $BQ - RQ = y_2 - y_1$

In the right-angled ΔARB,

$AB^2 = AR^2 + BR^2 \quad \Rightarrow d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\Rightarrow d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ie. $d^2 = (\text{difference of } x\text{-coordinates})^2 + (\text{difference of } y\text{-coordinates})^2$

$\Rightarrow d = \sqrt{(\text{diff. of } x\text{-coordinates})^2 + \text{diff. of } y\text{-coordinates}}$

Therefore, distance between A and B is

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 1:– The distance between points $(-3, 4)$ and $(2, -8)$

Let, $x_1 = -3$, $x_2 = 2$; $y_1 = 4$, $y_2 = -8$

$= \sqrt{(-3-2)^2 + (4-(-8))^2} = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ units

Ms. Reena

Example 2 Find the distance between the points A(3,0), B(4,5)

Solution:- Here, $x_1 = 3$, $y_1 = 0$ and $x_2 = 4$, $y_2 = 5$

Now, by using distance formula, we get

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4-3)^2 + (5-0)^2} = \sqrt{1+25} = \sqrt{26} \text{ Ans.}$$

Example 3:- Find the value of y for which the distance between the points P(2, −3) and Q(10, y) is 10 units.

Solution:- Given P(2, −3), Q(10, y) and PQ = 10

Now, by distance formula, we get

$$PQ = 10 \Rightarrow \sqrt{(10-2)^2 + (y-(-3))^2} = 10$$

$$\Rightarrow 64 + (y+3)^2 = 100$$

$$\Rightarrow (y+3)^2 = 36 \Rightarrow y+3 = \pm 6$$

$\Rightarrow y+3 = 6$ and $y+3 = -6$

$\Rightarrow y = 3$ and $y = -9$

Hence, the values of y are 3, −9

Important Note:-

* The distance of the point (x, y) from the origin $(0,0) = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2+y^2}$
* We can also write $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$ or $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ Both are correct.

* To prove that a given quadrilateral is a
  (i) Square → show that all sides equal and diagonals are also equal.
  (ii) Rhombus → show that all the sides are equal
  (iii) Rectangle → opposite sides equal and both diagonals are also equal.

Ms. Reena

(iv) Parallelogram → show that both the pairs of opposite sides are equal.
OR diagonals bisect each other.

* To prove that a triangle is
(i) Isosceles triangle → show any two sides equal
(ii) Equilateral triangle → show all sides are equal
(iii) a right angled triangle → show the sum of square of two sides is equal to the square of the third side.
* If points A, B, C are collinear, then sum of the distance between any two pairs of points is equal to distance between the remaining pairs of points.
* The coordinates of any point on x-axis is $(x, 0)$
* The coordinates of any point on y-axis is $(0, y)$

Example 4:- Show that the points $(6,9)$, $(0,1)$ and $(-6,-7)$ are collinear.

Solution:- Let $A(6,9)$, $B(0,1)$ and $C(-6,-7)$ be the given points

Then, $AB = \sqrt{(0-6)^2+(1-9)^2} = \sqrt{36+64} = \sqrt{100} = 10$ units

$BC = \sqrt{(-6-0)^2+(-7-1)^2} = \sqrt{36+64} = \sqrt{100} = 10$ units

$AC = \sqrt{(-6-6)^2+(-7-9)^2} = \sqrt{144+256} = \sqrt{400} = 20$ units

$\Rightarrow AC = AB + BC$

Hence, A, B, C are collinear.

Example 5:- Show that the points $(1,7)$, $(4,2)$, $(-1,-1)$ and $(-4,4)$ are vertices of a square

Solution:- Let $A(1,7)$, $B(4,2)$, $C(-1,-1)$ and $D(-4,4)$ be the given points.

$AB = \sqrt{(4-1)^2+(2-7)^2} = \sqrt{9+25} = \sqrt{34}$ (contd...)

Ms. Reena

$BC = \sqrt{(-1-4)^2 + (-1-2)^2} = \sqrt{25+9} = \sqrt{34}$

$CD = \sqrt{(-1+4)^2 + (4+1)^2} = \sqrt{9+25} = \sqrt{34}$

$DA = \sqrt{(-4-1)^2 + (4-7)^2} = \sqrt{25+9} = \sqrt{34}$

$\Rightarrow AB = BC = CD = DA$

$\Rightarrow$ all the four sides are equal.

Also, $AC = \sqrt{(-1-1)^2 + (-1-7)^2} = \sqrt{4+64} = \sqrt{68}$

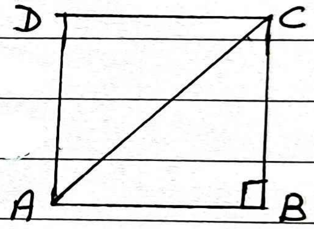
$BD = \sqrt{(-4-4)^2 + (4-2)^2} = \sqrt{64+4} = \sqrt{68}$

$\Rightarrow AC = BD$ $\Rightarrow$ both the diagonals are equal.

Hence, the given points are the vertices of a square.

Altenatively,

Find all the four sides AB, BC, CD, DA and one diagonal, say AC, as above



Show that $AB = BC = CD = DA$

Here, $AB^2 + BC^2 = 34 + 34 = 68$

$= AC^2$

Therefore, by converse of Pythagoras theorem, $\angle B = 90^\circ$.

Thus, all the four sides are equal and one angle is 90°

Hence, ABCD is a square.

-(Last page)