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Chapter - 5 Simultaneous Linear Equations

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Linear Equations in two variables:-

An equation of the form $ax+by+c=0$, where a, b, c are real numbers (constant) $a \neq 0$, $b \neq 0$ is called a linear equation in two variables x and y

Example:- (i) $4x - 3y = 7$ (ii) $0.4x + 0.3y = 2.7$

Every linear equation in two variables has an unlimited number of solutions.

Example 1:- $x + y - 3 = 0$

Here, $(x=0, y=3)$, $(x=1, y=2)$, $(x=2, y=1)$ etc. are all solutions of the equation $x+y-3=0$

Example 2:- Check, $x=4$, $y=1$ is a solution of the equation $2x + 3y = 11$

Solution:- Putting $x=4$ and $y=1$, we get

$$L.H.S. = 2 \times 4 + 3 \times 1 = 8 + 3 = 11 = R.H.S.$$

Here, L.H.S = R.H.S so, $x=4, y=1$ is a solution of the given equation.

Simultaneous Linear Equations in two variables:-

Two linear equations in two variables taken together are called simultaneous linear equations. The solution of system of simultaneous linear equation is the ordered pair (x,y) which satisfies both the linear equations.

The general form:- $$\left.\begin{array}{l} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{array}\right\}$$

Methods of Solving Simultaneous Linear Equations

I Substitution method

II Elimination method

III Cross - multiplication method

I SUBSTITUTION METHOD

Steps:-

1) Express y in terms of x from one of the given equations.
2) Substitute this value of y in the other equation to obtain a linear equation in x. Solve it for 'x.'
3) Substitute the value of x in the relation taken in step 1 and obtain the value of 'y'

Note:- We may interchange the role of x and y in the above method.

Example:- Solve the following system of linear equations
$$4x-3y=8\ ;\ x-2y=-3$$

Solution:- The given equations are

$4x-3y=8$ .... (i) $\qquad x-2y=-3$ .... (ii)

We can solve either equation for either variable. But to avoid fractions, we solve the second equation for x,

So, from equation (ii), we get $x = 2y-3$

Substituting this value of x in equation (i), we get

$4(2y-3)-3y=8 \Rightarrow 8y-12-3y=8 \Rightarrow 5y=20$

$\Rightarrow y=4$

Now, substituting the value of $y=4$ in equation (ii), we get

$x-2\times 4=-3 \Rightarrow x-8=-3 \Rightarrow x=5$

Hence, the solution is $x=5, y=4$

Note:- The solution may be checked in both the original equations.

II ELIMINATION METHOD

Steps:-

1) Multiply the given equations by suitable numbers so as to make the coefficients of one of the unknowns, numerically equal.
2) Add the new equations, if the numerically equal coefficients are opposite in sign, otherwise, subtract them.

contd...

3) The resulting equation is linear in one unknown. Solve it to obtain the value of one of the unknowns.

4) Substitute the value of this unknown in any of the given equations. solve it to get the value of the other unknown.

Example:- Solve the following system of linear equation

$$4x - 18 = 3y \; ; \; 6x + 7y - 4 = 0$$

Solution:- The given equations are :-

$$4x - 3y = 18 \text{ ------ (i)} \qquad 6x + 7y = 4 \text{ ------ (ii)}$$

Here, coefficient of x in both equations are 4 and 6

L.C.M of 4 and 6 is 12

So, we multiply equation (i) by 3 and equation (ii) by 2,

$$\Rightarrow [4x - 3y = 18] \times 3 \Rightarrow 12x - 9y = 54 \text{ ------ (iii)}$$

$$\text{and } [6x + 7y = 4] \times 2 \Rightarrow 12x + 14y = 8 \text{ ------ (iv)}$$

$$\begin{array}{r} - \quad - \quad - \\ \hline -23y = 46 \end{array}$$

Subtracting equation (iv) from equation (iii), we get

$$-23y = 46 \Rightarrow y = -2$$

Now, substituting $y = -2$ in equation (i), we get

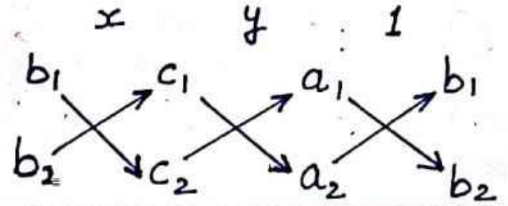
$$4x - 3 \times (-2) = 18 \Rightarrow 4x + 6 = 18 \Rightarrow 4x = 12 \Rightarrow x = 3$$

Hence, $x = 3$ and $y = -2$ is the solution of the given equations.

Note: The above example can be solved by making coefficient of y same (equal)

III CROSS MULTIPLICATION METHOD

Let the system of simultaneous linear equations be $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$



Here, the down arrows (↘) show the term with a plus sign and up arrows (↗) show the term with a negative sign.

(contd...)

The solution is given by

$$\frac{x}{b_1 \times c_2 - b_2 \times c_1} = \frac{y}{c_1 \times a_2 - c_2 \times a_1} = \frac{1}{a_1 \times b_2 - a_2 \times b_1}$$

$$\Rightarrow x = \frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1}, \quad y = \frac{c_1 a_2 - c_2 a_1}{a_1 b_2 - a_2 b_1}$$

Note :- The values of x and y can be obtained only when $a_1 b_2 - a_2 b_1 \neq 0$ i.e. when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Example: $x + 4y - 7 = 0$ and $2x + 8y + 2 = 0$

Here, $\frac{a_1}{a_2} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{4}{8} = \frac{1}{2} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

So, cross multiplication method is not applicable to the system of equations $x + 4y - 7 = 0$; $2x + 8y + 2 = 0$

Example :- Solve the following pair of linear equations by cross multiplication method :-

$2x + y = 5$; $3x + 2y = 8$

Solution:- The given equations can be written as

$2x + y - 5 = 0$ and $3x + 2y - 8 = 0$

Here, $a_1 = 2$, $b_1 = 1$, $c_1 = -5$ and $a_2 = 3$, $b_2 = 2$, $c_2 = -8$

Now, by cross multiplication method, we have

$$\begin{matrix} & x & & y & & 1 & \\ 1 & & -5 & & 2 & & 1 \\ 2 & & -8 & & 3 & & 2 \end{matrix} \Rightarrow \frac{x}{1 \times (-8) - 2 \times (-5)} = \frac{y}{(-5) \times 3 - (-8) \times 2} = \frac{1}{2 \times 2 - 3 \times 1}$$

$$\Rightarrow \frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3} \Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{2} = 1 \text{ and } \frac{y}{1} = 1 \Rightarrow x = 2 \text{ and } y = 1$$

Equations reducible to pair of linear Equations

Here we shall find solutions of such pairs of equations in two variables which are not linear but can be reduced to linear equations in two variables by making some suitable substitutions.

Example 1:- $\frac{1}{2x} + \frac{1}{3y} = 2$; $\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

Solution :- Substituting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in the given equation, we get $\frac{1}{2}u + \frac{1}{3}v = 2$ and $\frac{1}{3}u + \frac{1}{2}v = \frac{13}{6}$

$\Rightarrow 3u + 2v = 12$ and $2u + 3v = 13$ [By taking L.C.M]

Now, any of the three method discussed can be used for solving the given system of simultaneous linear equations unless a specific method is asked for.

Let us solve the given equation by Elimination Method

Given, $3u + 2v = 12$ ---- (i) and $2u + 3v = 13$ ---- (ii)

Here, coefficient of u in both equations are 3 and 2

L.C.M of 3 and 2 is 6

$\Rightarrow [3u + 2v = 12] \times 2 \Rightarrow 6u + 4v = 24$ ------- (iii)

and $[2u + 3v = 13] \times 3 \Rightarrow 6u + 9v = 39$ ------- (iv)

$$\begin{array}{r} 6u + 4v = 24 \\ \underline{\overset{-}{6u} + \overset{-}{9v} = \overset{-}{39}} \\ -5v = -15 \end{array}$$

Subtracting equation (iv) from equation (iii), we get

$-5v = -15 \Rightarrow v = 3 \Rightarrow \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$

Now, substituting $v = 3$ in equation (i), we get

$3u + 2 \times 3 = 12 \Rightarrow 3u = 12 - 6 \Rightarrow u = \frac{6}{3} \Rightarrow u = 2$

$\Rightarrow \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$

Hence, the solution of the given pair of equation is $x = \frac{1}{2}$ and $y = \frac{1}{3}$

Example 2:- $\frac{5}{x-1} + \frac{1}{y-2} = 2$; $\frac{6}{x-1} - \frac{3}{y-2} = 1$

Solution :- Substituting $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$ in the given equations, we get

$5p + q = 2$ ----- (i) and $6p - 3q = 1$ ----- (ii)

Now, by using substitution method, we get

From equation (i), $q = 2 - 5p$ ---- (iii)

(contd...)

Substituting this value of q in equation (ii), we get

$6p - 3(2-5p) = 1 \Rightarrow 6p - 6 + 15p = 1 \Rightarrow 21p = 7$

$\Rightarrow p = \frac{1}{3}$

Now, from equation (iii), $q = 2 - 5 \times \frac{1}{3} = 2 - \frac{5}{3} = \frac{1}{3}$

Therefore, $\frac{1}{x-1} = \frac{1}{3}$ and $\frac{1}{y-2} = \frac{1}{3} \Rightarrow x - 1 = 3$ and

$y - 2 = 3 \Rightarrow x = 4$ and $y = 5$

Hence, the solution of the given pair of equations is $x = 4$ and $y = 5$

Example 3:- $4x + 9y = 30xy$; $5y - 3x = xy$

Solution:- Here, $x = 0, y = 0$ is a solution of the given system of equations.

Now, when $x \neq 0, y \neq 0$, then dividing both sides of each equation by xy, we get

$\frac{4}{y} + \frac{9}{x} = 30$ ------ (i) and $\frac{5}{x} - \frac{3}{y} = 1$ -------- (ii)

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$, then egations (i) and (ii),

we get $9a + 4b = 30$ ----- (iii) and $5a - 3b = 1$ ----- (iv)

By Elimination Method, we have

$[9a + 4b = 30] \times 3 \Rightarrow 27a + 12b = 90$

$[5a - 3b = 1] \times 4 \Rightarrow + \; 20a - 12b = 4$

$47a = 94$

$\Rightarrow 47a = 94 \Rightarrow a = 2$

Now, substituting $a = 2$ in equation (iii), we get

$9 \times 2 + 4b = 30 \Rightarrow 4b = 12 \Rightarrow b = 3 \Rightarrow y = \frac{1}{3}$

Hence, $x = 0, y = 0$; $x = \frac{1}{2}, y = \frac{1}{3}$

EXTRA QUESTIONS

1. Solve the following systems of equations:

(i) $x + y = 6$
$x - y = 2$

(ii) $x + y = 7$
$5x + 12y = 7$

(iii) $3x + 2y = 10$
$4x - 3y = 19$

(iv) $3x + 4y = 5$
$6x - 4y = -14$

(v) $5x + 4y = 4$
$x - 12y = 20$

(vi) $5m - 5n = 12$
$2m + 9n = 20$

(vii) $17x + 10y = 118$
$15x - 6y = 30$

(viii) $3x + 4y = 25$
$5x - 6y = -9$

(ix) $x + 2y = \frac{3}{2}$
$2x + y = \frac{3}{2}$

(x) $\frac{x}{3} = \frac{y}{6}$
$\frac{4x}{3} + 2y = 4$

(xi) $x = -4y$
$\frac{x - y}{10} = \frac{x - 6}{2}$

(xii) $2x - \frac{y}{4} = 5$
$x + \frac{3}{4}y = -1$

(xiii) $x - 4 = 4(y + 2)$
$3(x - 2) = 2y + 20$

(xiv) $x + 2y = \frac{3}{2}$
$2x + y = 3$

(xv) $x - y = 0.9$
$\frac{11}{x + y} = 2.$

(xvi) $2x - y = 4.1$
$\frac{11}{2(x + y)} = 1$

2. Solve the following systems of simultaneous equations:

(i) $\frac{11}{x} - \frac{7}{y} = 1$
$\frac{9}{x} - \frac{4}{y} = 6$

(ii) $\frac{15}{x} + \frac{2}{y} = 17$
$\frac{1}{x} + \frac{1}{y} = \frac{36}{5}$

(iii) $2x - \frac{3}{y} = 9$
$3x + \frac{7}{y} = 2$

(iv) $\frac{7}{x} + \frac{8}{y} = 2$
$\frac{2}{x} + \frac{12}{y} = 20$

(v) $3(2x + y) = 7xy$
$3(x + 3y) = 11xy$

(vi) $8x - 3y = 5xy$
$6x - 5y = -2xy$

(vii) $\frac{3}{x + y} + \frac{2}{x - y} = 3$
$\frac{2}{x + y} + \frac{3}{x - y} = \frac{11}{3}$

(viii) $x + y = 7xy$
$\frac{2}{x} + \frac{3}{y} = 17.$

3. If $2x + y = 35$ and $3x + 4y = 65$, find the value of $\frac{x}{y}$.

4. If $2x + y = 23$ and $4x - y = 19$, find the value of $x - 3y$ and $5y - 2x$.

5. Solve: $4x + \frac{6}{y} = 15$ and $6x - \frac{8}{y} = 14$. Hence, find 'λ' if $y = \lambda x - 2$.

Solve each of the following systems of equations:

1. $3x + 2y = 8$
$x - 3y + 1 = 0$

2. $2x + y = 1$
$3x + 5y = 5$

3. $2x + 3y + 1 = 0$
$x + y = 0$

4. $4x + 3y = 65$
$x + 2y = 35$

5. $3x - 2y = 12$
$4x - 5y = 16$

6. $3x - 7y = 7$
$11x + 5y = 87$

7. $\frac{7}{2}x + 2y = 5$
$\frac{35}{2}x + 12y = 25$

8. $\frac{2}{x + y} + \frac{3}{x - y} = 1$
$\frac{8}{x + y} - \frac{7}{x - y} = \frac{5}{6}$

[Hint: Put $\frac{1}{x + y} = u$ and $\frac{1}{x - y} = v$.]

9. $\frac{2}{x} + \frac{3}{y} = 13$
$\frac{5}{x} - \frac{4}{y} = -2 \qquad (x \neq 0, y \neq 0)$

10. $\frac{5}{x + y} - \frac{2}{x - y} + 1 = 0$
$\frac{15}{x + y} + \frac{7}{x - y} = 0$

[Hint: Write $\frac{1}{x + y} = p$ and $\frac{1}{x - y} = q$.]