Tender Heart High School, Sector 33B, Chd.

Class : 9th
Subject : Mathematics

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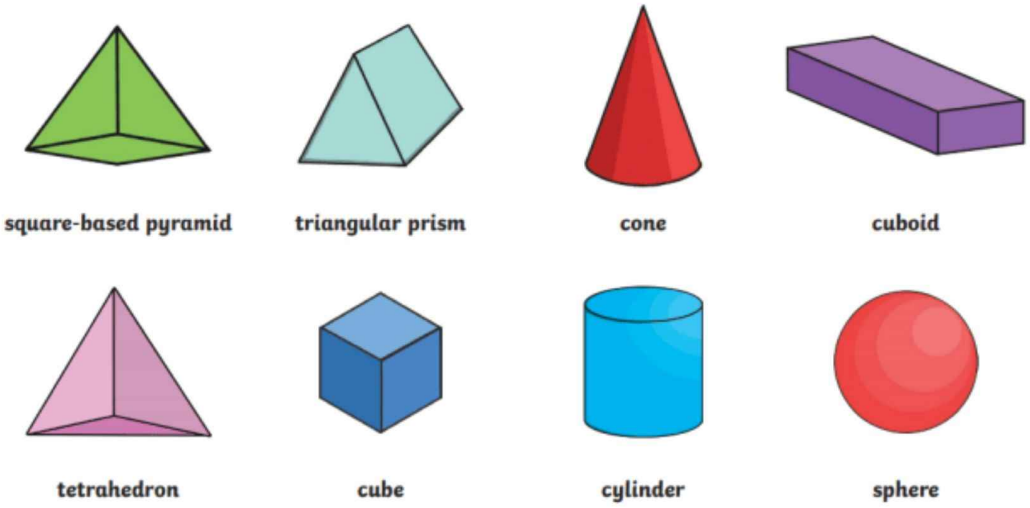
Ms Reena

Chapter - 19

Surface Area and Volume of 3-D Shapes



Students in the previous chapter we have discussed about two dimensional shapes that is area and perimeter of triangles, quadrilaterals and circles. Now, in this chapter we will discuss about surface area and volume of 3-D solids. that is,

Cube and cuboid including problems of type involving :-

1) Different Internal and External dimensions of the solid.
2) Cost
3) Concept of volume being equal to area of cross-section x height
4) Open / closed cubes / cuboids

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What is a solid shape?

Objects that occupy space are called solid shapes. A solid has three dimensions – length, breadth and height.

Some examples of solid shapes are cone, cuboid, sphere, cylinder, Cube... sphere has neither any edges nor vertices.

Solid figures are identified according to the features that are unique to each other type of solid. Specifically, we can observe the number of faces, edges and vertices, as well as the shape of the base. The flat surfaces of a solid figures are its faces.

Solid shapes are present everywhere around us. We live in a three-dimensional world. Every one of us has height, width and length. Examples in daily life are Dice, Ice cube, Funnel, Ice cream cone, Boxes, Book, Fridge, etc.

Volume of a solid:- The measurement of the space enclosed by a solid is called its volume.

Surface area of a solid:- The sum of the areas of the plane or curved surfaces (faces) of a solid is called its total surface area.

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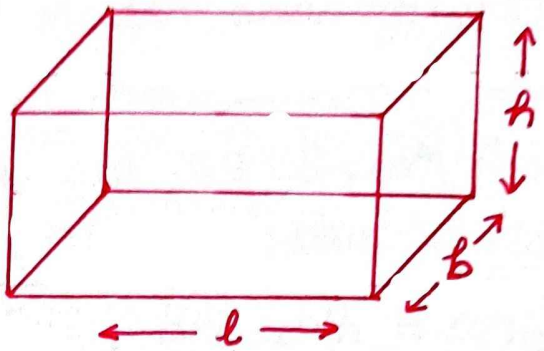
CUBOID



A rectangular solid bounded by six rectangular plane faces is called a cuboid.

Examples:- A match box, a chalk box, a tea-packet, a brick, a book, etc.

A cuboid has 6 rectangular faces, 12 edges and 8 vertices.

Volume of a Cuboid

= length × breadth × height

= $(l \times b \times h)$ cubic units

Diagonal of a Cuboid

The line joining opposite corners of a cuboid is called its diagonal.

A cuboid has four diagonals.

Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$ units

Total Surface Area of a Cuboid

= $2(lb + bh + lh)$ sq. units

Lateral Surface Area of a Cuboid

= $2[(l + b) \times h]$ sq. units that is $2(lh + bh)$

= Area of four walls $2h(l + b)$

Example 1:- The length, breadth and height of a rectangular solid are in the ratio 6:5:4. If the total surface area is 5328 cm^2, find the length, breadth and height of the solid.

Solution:- Let length = $6x$, breadth = $5cm$ and height = $4x$

$2(l \times b + b \times h + h \times l)$

Then, total surface area

$= [2(6x \times 5x + 5x \times 4x + 4x \times 6x)]\ cm^2$

$= 2(30x^2 + 20x^2 + 24x^2)\ cm^2$

$= 148x^2\ cm^2$

$\Rightarrow 148x^2\ cm^2 = 5328 \Rightarrow x^2 = \frac{5328}{148} = 36$

$\Rightarrow x = 6$

Hence, length = 36cm, breadth = 30cm and height = 24cm

Example 2:- A cuboid has length, breadth and diagonal as 4m, 3m and 13m respectively. Find its volume.

Solution:- Let the height of the cuboid be 'h' m

Here, $l = 4m$ and $b = 3m$

As the length of diagonal of a cuboid $= \sqrt{l^2+b^2+h^2}$

Therefore, $\sqrt{4^2 + 3^2 + h^2} = 13$

$\Rightarrow 16 + 9 + h^2 = 169 \Rightarrow h^2 = 144 \Rightarrow h = 12$

Height of cuboid = 12m

Volume of cuboid = $l \times b \times h = 4 \times 3 \times 12$

$= 144\ m^3$

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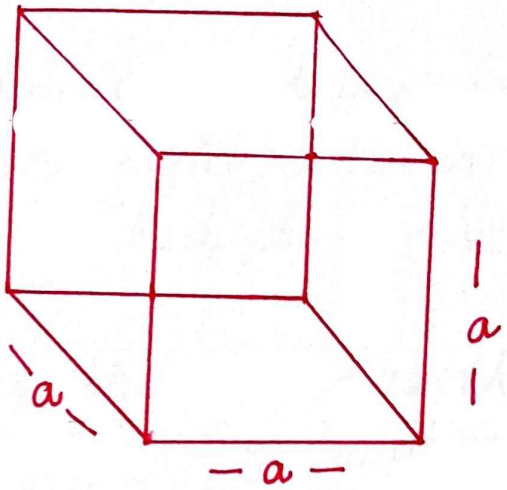
CUBE



A cuboid whose length, breadth and height are all equal is called a cube.

Examples:- Ice cubes, Sugar cubes, Dice etc.

Each edge of a cube is called its side. Let us consider a cube of edge = a units. Then,

Volume of cube = a^3 cubic units.

Diagonal of the cube = $a\sqrt{3}$ units.

Total surface Area of the cube = $6a^2$ sq.unit

Lateral surface Area = $4a^2$ sq. units.

Note:-

1) The capacity of a container = its internal volume

2) The volume of material in a hollow body = its external volume − its internal volume.

3) If the external dimensions (length, bredth and height) of a box are $l, b,$ and if each side is of thickness x, the the internal dimensions of the

(i) closed box are $l-2x, b-2x, h-2x$

(ii) open box are $l-2x, b-2x, h-x$

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CROSS-SECTION

If a cut is made through a solid perpendicular to its length (breadth or height), then the surface so obtained is called its cross-section

If the surface made by the cut has the same shape and size at every point of its length (breadth or height), then it is called a uniform cross-section.

(i) Volume = area of cross-section x length

(ii) lateral surface area
= perimeter of cross-section x length

Example 3:- A solid piece of metal, cuboidal in shape, with dimensions 24 cm, 18 cm and 4 cm is recast into a cube. Calculate the lateral surface area of the cube.

Solution:- Volume of the cuboid = $24 \times 18 \times 4$

Let 'a' side of the cube be a cm, then its volume = a^3 cm^3

Since metal of the cuboid is to be recasted into a cube,

Volume of cube = volume of cuboid

$\Rightarrow a^3 = 24 \times 18 \times 4$

$= 24 \times 72$ or $12 \times 144 = (12)^3$

$\Rightarrow a = 12$

Lateral surface of the cube $= 4a^2 = 576$ cm^2

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Example 4:- To construct a wall 25 m long, 0.3m thick and 6m high, bricks of dimensions 25 cm x 15cm x 10cm, each are used. If mortar occupies $\frac{1}{10}$ th of the volume of the wall, find the number of bricks used.

Solution:- Length of wall = 25m = 2500cm, thickness = 0.3m = 30cm and height = 6m = 600cm

Volume of wall = $l \times b \times h = 2500 \times 30 \times 600$ cm^3

Dimensions of each brick are 25cm x 15 x 10

Therefore, Volume of each brick = 25x15x10

Since $\frac{1}{10}$ th of volume of wall is occupied by morta..

So, the volume of wall occupied by bricks

$= \left(1 - \frac{1}{10}\right)$ th of volume of wall

$= \frac{9}{10} \times 2500 \times 30 \times 600 = (2500 \times 27 \times 600)\ cm^3$

Therefore, the number of bricks required to construct the wall

$= \frac{\text{Volume of wall occupied by bricks}}{\text{Volume of one brick}}$

$= \frac{2500 \times 27 \times 600}{25 \times 15 \times 10}$

$= 10800$

Example 5 :- A tank measuring 15 m long, 10 m broad and 4 m deep is full of water. The water is pumped into another tank which is 20 m long and 15 m broad. Find the depth of water in the new tank.

Sol. :- Given dimensions of 1st tank are 15 m long, 10 m broad and 4 m deep

Let the depth of water in new tank be 'h'

Now, volume of 1st tank = volume of new tank

$\Rightarrow 15 \times 10 \times 4 = 20 \times 15 \times h$

$\Rightarrow h = \frac{15 \times 10 \times 4}{20 \times 15} = 2 \text{ m}$

So, depth of water in new tank = 2 m

Example 6 The volume of a cube is 1331 cm^3. Find its total surface area.

Sol. :- Volume of cube = $(\text{side})^3$

$\Rightarrow (\text{side})^3 = 1331 = (11)^3$

So, side of a cube = 11 cm

Total surface area of cube = $6 \times (\text{side})^2$

$= 6 \times 11 \times 11$

$= 726 \text{ cm}^2$

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Example 7 The edges of three cubes of metal are 3cm, 4cm and 5cm. They are melted and formed into a single cube. Find the edge of the new cube.

Sol.:- We know that,

Volume of cube = $(\text{side})^3$

Now, volume of three small cubes

= Volume of a single cube

$\Rightarrow (3)^3 + (4)^3 + (5)^3 = (\text{side})^3$

$\Rightarrow 27 + 64 + 125 = 216 = (6)^3$

So, edge of new cube = 6cm

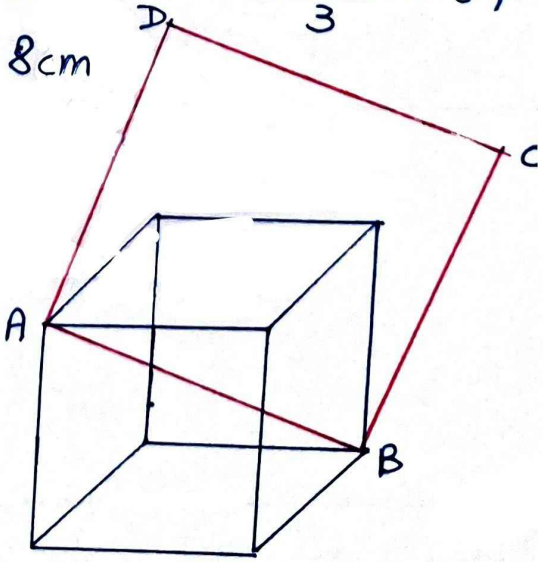
Example 8 The square on the diagonal of a cube has an area of 192 cm^2. Calculate:-
(i) edge of the cube (ii) total surface area.

Solution:- Diagonal of a cube = side $\sqrt{3}$ or $a\sqrt{3}$

$\Rightarrow (\text{side}\sqrt{3})^2 = 192 \Rightarrow (\text{side})^2 = \frac{192}{3} = 64$

$\Rightarrow$ side or edge of cube = 8cm

Here AB is the diagonal of a cube. And ABCD is a square on the diagonal of a cube.



Total surface area of a cube = $4 \times (8)^2$

= $256\ cm^2$

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Ques 9 :- The area of a playground is 4800 sq m. Find the cost of covering it with gravel 1cm deep if the gravel costs ₹ 4.80 per cubic metre.

Sol.:- Area of playground = $4800\ m^2$ [1m = 100cm]

$= 48000000\ cm^2$

Depth of gravel = 1cm = $\frac{1}{100}$ m

Volume of gravel on playground = 48000000×1

$= 48000000\ cm^3$

$= 48\ m^3$

Rate of gravel = ₹ 4.80 per cubic metre

So, cost = 48×4.80 = ₹ 230.40

Ques 10 :- A wall of dimensions 25 m by 0.5 m by 2 m is to be built by using bricks of dimensions 20 cm by 10 cm by 6 cm. Find the number of bricks required. Give your answer to the nearest hundred

Sol.:- Number of bricks

$$= \frac{\text{Volume of wall}}{\text{Volume of 1 brick}} \qquad l \times b \times h$$

$$= \frac{2500\ cm \times 50\ cm \times 200\ cm}{20\ cm \times 10\ cm \times 6\ cm}$$

$$= 20833.33$$

$$= 20900 \text{ bricks}$$

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