Tender Heart High School, Sector 33B, Chd.

Class 10th Maths Date: 20.05.2024

Ms. Reena

Chapter 10 Arithmetic Progression

Introduction :- In this chapter we will do the following:- 1) Finding General term of progressions
2) Finding sum of their first 'n' terms
3) Simple Applications.

Everywhere around us, in all walks of our life, - in nature, architecture, rangolis, quilts, numbers etc. we observe patterns.

Consider the following sets of number :-

1) 2, 7, 12, 17, - - - -
2) 1, 2, 4, 8, 16, - - -
3) 22, 18, 14, 10, 6, - - -
4) 1, 3, 6, 4, 2, - - -

Here, the sets of numbers are arranged in a specific order and according to a definite rule, i.e.

1) Increasing order → adding 5 to each term.
2) Increasing order → multiplying each term by 2
3) Decreasing order → adding 4 and (−4)
4) Does not follow any order or rule.

So, the sets of numbers in (1), (2) and (3) are called sequences. Thus, a sequence is a set of numbers specified in a definite order by some assigned rule.

→ Each element of the set is called a term.

→ A finite sequence is that which ends or has a last term. e.g. 4, 9, 14, 19, 24 is a finite sequence.

→ An Infinite sequence is one which has no last term.
e.g. 40, 35, 30, 25, 20, 15, 10, 5, 0, −5, - -

→ A series is defined as the sum of the terms of a sequence.
e.g. (i) $2+7+12+17+\dots$
(ii) $22+18+14+10+6+\dots$

Class 10, Mathematics
Chapter – 10

Ms. Reena

Arithmetic Progression (AP)

→ A sequence in which its terms continually increase or decrease by the same number is called an arithmetic progression.

→ The fixed number by which they increase or decrease is called the common difference of the A.P. denoted by 'd'

→ The first term of an A.P. is generally denoted by 'a'

Example 1:- 11, 15, 19, 23, 27, ---

Here, first term $= a = 11$ and

common difference $= d = 15-11 = 4$ or $19-15 = 4$ ...

General Term (n^{th} term) of an A.P.

If 'a' is the first term and 'd' is the common difference of an A.P., then

First term $= a = a+(1-1)d$

Second term $= a+d = a+(2-1)d$

Third term $= a+2d = a+(3-1)d$ and so on.

So, n^{th} term $= a+(n-1)d$

→ First, second, third, --- term of an A.P. are respectively denoted by $a_1, a_2, a_3, ----$.

or by $t_1, t_2, t_3, ----$

→ The general form of A.P. is

$a, a+d, a+2d, a+3d, a+4d, ---$

where, $a_1 = a$, $a_2 = a+d$, $a_3 = a+3d$ and so on

and $a_n = a+(n-1)d$

→ last term, $l = a+(n-1)d$ when A.P. is finite

Middle term(s) of a finite A.P.

(i) $\left(\frac{n+1}{2}\right)$ th term when 'n' is odd

(ii) $\left(\frac{n}{2}\right)$ th and $\left(\frac{n}{2}+1\right)^{th}$ term when 'n' is even

Ms. Reena

Example 2:- Check which of the following sequences are A.P.'s

(i) 11, 22, 33, 44, ...

Here, first term, $a = 11$ and

common difference, d = any term − preceding term

Let $a_1 = 11$, $a_2 = 22$, $a_3 = 33$ and $a_4 = 44$

so, $d = a_2 - a_1 = 22 - 11 = 11$, $a_3 - a_2 = 33 - 22 = 11$,

$a_4 - a_3 = 44 - 33 = 11$ etc.

Since, common difference = 11 in all cases. ie. same

Hence, the given list of numbers forms an A.P.

(ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

Here, first term, $a = \frac{1}{2}$ and

common difference, $d = a_2 - a_1 = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6}$

$a_3 - a_2 = \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = -\frac{1}{12}$

$\Rightarrow a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers does not form an A.P.

(iii) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

$= \sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}, \ldots$

Here, $d = a_2 - a_1 = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

$a_3 - a_2 = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$ and $a_4 - a_3 = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$

Since, $d = \sqrt{3}$ in all cases

Hence, the given list of numbers forms an A.P.

(iv) $2, 2^2, 2^3, 2^4, \ldots = 2, 4, 8, 16, \ldots$

Here, $d = a_2 - a_1 = 4 - 2 = 2$, $a_3 - a_2 = 8 - 4 = 4$

$\Rightarrow a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers does not form an A.P.

Example 3 Find the next two terms of given A.P.

$\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, ---$

Solution:- Since the given list of numbers are in A.P. Therefore, $a = \frac{3}{2}$

and common difference, $d = \frac{1}{2} - \frac{3}{2} = \frac{1-3}{2} = \frac{-2}{2} = -1$

Here, $a_1 = \frac{3}{2}$, $a_2 = \frac{1}{2}$, $a_3 = -\frac{1}{2}$, $a_4 = -\frac{3}{2}$

$a_5 = -\frac{3}{2} + (-1) = \frac{-3-2}{2} = -\frac{5}{2}$

$a_6 = -\frac{5}{2} + (-1) = \frac{-5-2}{2} = -\frac{7}{2}$

So, the next two terms are $-\frac{5}{2}$ and $-\frac{7}{2}$

Example 4 Find the 7^{th}, 40^{th} and n^{th} term of the A.P. 5, 1, −3, −7, ...

Solution:- $a = 5$, $d = 1 - 5 = -4$

We know that, $a_n = a + (n-1)d$

Put $a = 5$ and $d = -4$, we get

$a_n = 5 + (n-1)(-4) = 5 - 4n + 4 = 9 - 4n$

$\Rightarrow$ n^{th} term $= 9 - 4n$

Now, 7^{th} term i.e. $a_7 = 5 + (7-1)(-4) = 5 - 24 = -19$

and 40^{th} term ie. $a_{40} = 5 + (40-1)(-4) = 5 - 156 = -151$

So, n^{th} term $= 9 - 4n$, 7^{th} term $= -19$, and 40^{th} term $= -151$

Note:- In the above example since n^{th} term i.e. $a_n = 9 - 4n$

$\Rightarrow a_1 = 9 - 4 \times 1 = 5$, $a_2 = 9 - 4 \times 2 = 1$, $a_3 = 9 - 4 \times 3 = -3$ and so on

Example 5: Which term of the A.P. 21, 18, 15, ... is −81?

Solution:- Here, first term, $a = 21$ and common difference, $d = 18 - 21 = -3$

Let nth term of the given A.P. be −81, then $a_n = -81 \Rightarrow a + (n-1)d = -81$

Now, putting value of $a = 21$ and $d = -3$, we get

$21 + (n-1)(-3) = -81 \Rightarrow 21 - 3n + 3 = -81$

$\Rightarrow -3n = -81 - 21 - 3 \Rightarrow -3n = -105 \Rightarrow n = 35$

Hence, −81 is the 35 th term

Example 6: Write an A.P. whose 3rd term is 5 and the 7th term is 9.

Solution:- $a_3 = a + (3-1)d \Rightarrow a + 2d = 5$ ---- (i)

$a_7 = a + (7-1)d \Rightarrow a + 6d = 9$ ---- (ii)

Subtracting (i) from (ii), we get

$$\begin{array}{r} a + 2d = 5 \\ -\ a + 6d = 9 \\ \hline 4d = 4 \end{array}$$

$\Rightarrow d = 1$ and $a = 3$

Hence, $a_1 = a + (1-1)d = 3$

$a_2 = a + (2-1)d = 3 + 1 = 4$

$a_3 = a + (3-1)d = 3 + 2 \times 1 = 5$

So, the required A.P. is. 3, 4, 5, 6, ...

Example 7: How many multiples of 4 lie between 10 and 250?

Solution:- Multiples of 4 which lie between 10 and 250 are 12, 16, 20, ..., 248

Here, $a = 12$, $d = 16 - 12 = 4$ and $a_n = 248$

Now, using $a_n = a + (n-1)d$

$\Rightarrow 248 = 12 + (n-1)(4) \Rightarrow 4n - 4 = 236$

$\Rightarrow n - 1 = 59 \Rightarrow n = 60$ So, there are 60 multiples

Class 10, Mathematics

Ms. Reena

Example 8:- If $(k-3)$, $(2k+1)$ and $(4k+3)$ are three consecutive terms of an A.P., find the value of 'k'.

Solution:- Let $a_1 = k-3$, $a_2 = 2k+1$, $a_3 = 4k+3$

Now, common difference, $d = a_2 - a_1 = a_3 - a_2$

$\Rightarrow (2k+1)-(k-3) = (4k+3)-(2k+1)$

$\Rightarrow 2k+1-k+3 = 4k+3-2k-1$

$\Rightarrow k+4 = 2k+2 \quad \Rightarrow 2 = k$

Hence, the value of k is 2.

Example 9:- Which term of the A.P. 120, 116, 112, ... is its first negative term?

Solution:- Here, $a = 120$, $d = 116-120 = -4$

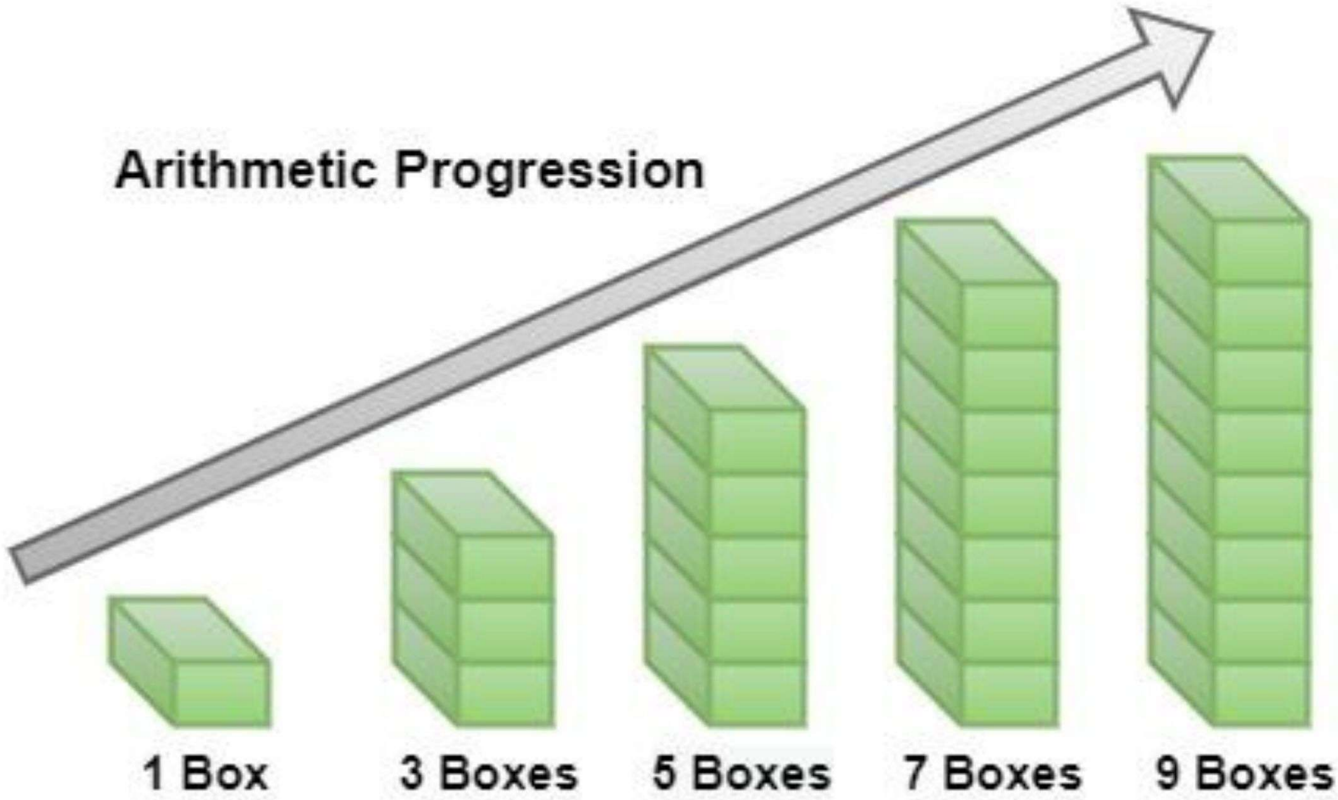
We know that $a_n = a+(n-1)d$

Now, $120+(n-1)(-4) < 0$ is the first negative term

$\Rightarrow 120-4n+4 < 0$

$\Rightarrow 124-4n < 0 \Rightarrow 124 < 4n \Rightarrow n > 31$

$\Rightarrow n = 32$ i.e. 32nd term is the first negative term.



Sum of first 'n' terms of an A.P.

Let us consider the problem given to Gauss (One of the three greatest mathematicians of all time, alongwith Newton and Archimedes) when he was just 10 years old.
He was asked to find the sum of natural numbers from 1 to 100. He immediately replied that the sum is 5050

His solution was as follows:-

Sum$(S) = 1 + 2 + 3 + \cdots + 98 + 99 + 100$

and then he reversed the order of numbers and wrote Sum$(S) = 100 + 99 + 98 + \cdots + 3 + 2 + 1$

On adding these two, he got

$$2S = 101 + 101 + 101 + \cdots + 101 + 101 + 101 \text{ (100 times)}$$

$$\Rightarrow 2S = 100 \times 101 \Rightarrow S = \frac{100 \times 101}{2} = 50 \times 101$$

$$\Rightarrow S = 5050$$

→ We shall use the same technique to find the sum of first 'n' terms of an A.P.

→ Let 'a' be the first term and 'd' be the common difference of an A.P. consisting of 'n' terms and 'l' be its last term, then the A.P. is :-

$$a, a+d, a+2d, \cdots, l$$

where, $l = a + (n-1)d$.

If S_n (or S) denotes the sum of first 'n' terms of the A.P., then

$$S_n = a + (a+d) + (a+2d) + \cdots + (l-2d) + (l-d) + l \quad \text{(i)}$$

Writing the terms of the A.P. in reverse order, we have $S_n = l + (l-d) + (l-2d) + \cdots + (a+d) + a$ (ii)

On adding these two, we get

Ms. Reena

$2S_n = (a+l) + (a+l) + \dots + (a+l)$ n times

$\Rightarrow 2S_n = n(a+l)$ $\Rightarrow S_n = \frac{n}{2}(a+l)$ ----(iii)

Writing the value of $l = a+(n-1)d$ in (iii), we get

$S_n = \frac{n}{2}[a + a + (n-1)d] = \frac{n}{2}[2a + (n-1)d]$

Thus, $S_n = \frac{n}{2}[2a+(n-1)d]$ or $S_n = \frac{n}{2}(a+l)$

Note: The formula $S_n = \frac{n}{2}[2a+(n-1)d]$ for the sum of first 'n' terms of an A.P. involves four quantities S_n, a, d and n. If we know any three of them, we can find the fourth quantity.

* In an A.P., $a_n = S_n - S_{n-1}$
  i.e. a_n = sum of first 'n' terms − sum of first $(n-1)$ terms
* Sum of first 'n' natural numbers
  The first 'n' natural numbers i.e. $1, 2, 3, 4, \dots, n$ form an A.P. with $a = 1$, $d = 1$ and $l = n$
  Using the formula, $S_n = \frac{n}{2}(a+l)$, we get

  sum of first 'n' natural numbers $= \frac{n}{2}(1+n)$

  $\Rightarrow S_n = \frac{n(n+1)}{2}$

Example 1:- $-37, -33, -29, \dots$ to 12 terms

Here, $a = -37$, $d = -33 - (-37) = 4$ and $n = 12$

Now, Sum$(S_n) = \frac{n}{n}[2a+(n-1)d]$

$\Rightarrow S_{12} = \frac{12}{2}[2(-37) + (12-1)\times 4] = 6(-74+44)$

$= 6 \times (-30) = -180$

Class 10, Mathematics,

Ms. Reena

Example 2:- Find the sums given below:-

$$7 + 10\frac{1}{2} + 14 + \cdots + 84$$

Solution:- The given numbers form an A.P. with

$a = 7$ and $d = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$

Let n^{th} term be 84, then $a_n = a + (n-1)d$

$\Rightarrow 84 = 7 + (n-1) \times \frac{7}{2} \quad \Rightarrow 12 = 1 + \frac{n-1}{2}$

$\Rightarrow \frac{n-1}{2} = 11 \quad \Rightarrow n - 1 = 22 \quad \Rightarrow n = 23$

Now, sum $(S_n) = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{23} = \frac{23}{2}\left[2 \times 7 + (23-1) \times \frac{7}{2}\right] = \frac{23}{2}(14 + 77)$

$= \frac{23 \times 91}{2} = \frac{2093}{2} = 1046\frac{1}{2}$

Note:- In the above example since number of term i.e. 'n' is not given. so, first we apply formula $a_n = a + (n-1)d$ to get value of 'n' and then $S_n = \frac{n}{2}[2a + (n-1)d]$

Example 3:- Find the sum of first 24 terms of the list of numbers whose n^{th} term is given by

$a_n = 3 + 2n$

Solution:- Given, $a_n = 3 + 2n$

Putting $n = 1, 2, 3, 4, \ldots$, we get

$a_1 = 3 + 2 \times 1 = 5$, $a_2 = 3 + 2 \times 2 = 7$

$a_3 = 3 + 2 \times 3 = 9$, $a_4 = 3 + 4 \times 2 = 11, \ldots$

Thus, the list of numbers is $5, 7, 9, 11, \ldots$

Now, $a_{n+1} - a_n$ i.e. $d = 7 - 5 = 2$

Using formula, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$S_{24} = \frac{24}{2}[2 \times 5 + 23 \times 2] = 12 \times 56 = 672.$

Class 10, Mathematics

Ms. Reena

Example 4:- In an A.P., if $S_n = n(4n+1)$, find the A.P.

Solution:- In an A.P., given $S_n = n(4n+1)$ or $4n^2+n$

We know that $a_n = S_n - S_{n-1}$

Now, $S_1 = 1(4\times1+1) = 5$, $S_2 = 2(4\times2+1) = 18$

$S_3 = 3(4\times3+1) = 39$, $S_4 = 4(4\times4+1) = 68$

$\Rightarrow a_1 = S_1 = 5$, $a_2 = S_2 - S_1 = 18-5 = 13$

$a_3 = S_3 - S_2 = 39-18 = 21$, $a_4 = S_4 - S_3 = 68-39 = 29$

Hence, the A.P. is 5, 13, 21, 29, ---

Summary of A.P.

1) The first, second, third, --- terms of A.P. are $a_1, a_2, a_3, \text{-----} a_n$ if $d = a_{n+1} - a_n$

i.e. $a, a+d, a+2d, a+3d, \text{---} a+(n-1)d$

2) $a_n = a+(n-1)d$ and if 'l' is the last term then, nth term from end $= l-(n-1)d$

3) Middle term(s) of an A.P.

If 'n' is odd, then $\left(\frac{n+1}{2}\right)^{th}$ term

If 'n' is even, then $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ term

4) If three numbers are in A.P. ie. a, b and c then $2b = a+c$ or If x, y and z are in A.P, then $2y = x+z$

5) Sum of numbers in A.P. is given, then

(i) For 3 numbers $\rightarrow$ $a-d, a, a+d$

(ii) For 4 numbers $\rightarrow$ $a-3d, a-d, a+d, a+3d$

6) $S_n = \frac{n}{2}[2a+(n-1)d]$