Tender Heart High School, Sec. 33B, Chd.

Class : 10th | Date : 13th May 2024

Subject : Mathematics | Teacher : Ms. Reena

Chapter - 9 Matrices

Introduction :- A matrix is an ordered rectangular arrangement of numbers. The numbers are called the elements or the entries of the matrix. The horizontal lines of elements in the matrix is called rows, and the vertical lines of elements are called the columns.

* We represent a matrix by capital letter e.g. A, B, C, X, Y, Z etc.

Order of a Matrix

= Number of rows × Number of columns

$$A = \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 \\ \downarrow & \downarrow & \downarrow \end{matrix} \\ \begin{bmatrix} 3 & 4 & 7 \\ 0 & 8 & 9 \\ 2 & -5 & 3 \end{bmatrix} \end{array} \begin{matrix} \\ \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{matrix}$$

It has 3 rows and 3 columns.
Therefore its order is 3×3

Representation of Matrix

$A\ [a_{ij}]_{m \times n}$, where a_{ij} is the element corresponding to i^{th} row and j^{th} column.

Class 10th Matrices Ms. Reena

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$$ is a 2 × 2 matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times 2}$$ is 3 × 2 matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n} = [a_{ij}]_{m\times n}$$

TYPES OF MATRICES

NAME	DESCRIPTION	EXAMPLE
Rectangular matrix	No. of rows is not equal to no. of columns	$\begin{bmatrix} 6 & 2 & -1 \\ -2 & 0 & 5 \end{bmatrix}$
Square matrix	No. of rows is equal to no. of columns	$\begin{bmatrix} 2 & -1 & 3 \\ -2 & 0 & 1 \\ 1 & 2 & 4 \end{bmatrix}$
Diagonal matrix	Non-zero element in principal diagonal and zero in all other positions	$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$
Scalar matrix	Diagonal matrix in which all the elements on principal diagonal and same	$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Class 10th Maths

TYPES OF MATRICES

NAME	DESCRIPTION	EXAMPLE
Row matrix	A matrix with only 1 row	$\begin{bmatrix} 3 & 2 & 1 & -4 \end{bmatrix}$
Column matrix	A matrix with only I column	$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$
Identity matrix	Diagonal matrix having each diagonal element equal to one (I)	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Zero matrix	A matrix with all zero entries	$\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$

Add and subtract matrices

$$\begin{bmatrix} 8 & 5 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 9 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 5 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 9 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$$

Class 10th Maths

2 × 2 Matrix Multiplication

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \times \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1a_2 + b_1c_2 & a_1b_2 + b_1d_2 \\ c_1a_2 + d_1c_2 & c_1b_2 + d_1d_2 \end{bmatrix}$$

e.g. $\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 3 + 12 & 15 + 28 \\ 2 + 3 & 10 + 7 \end{bmatrix}$

Matrix 1 Matrix 2

$$= \begin{bmatrix} 15 & 43 \\ 5 & 17 \end{bmatrix}$$

Resultant Matrix

Rule For Matrix Multiplication

$$A \quad . \quad B \quad = \quad AB$$

$$m \times n \qquad n \times p \qquad m \times p$$

Equal

Dimensions of AB

Matrices

Ms. Reena

Q1: If $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, is the product AB possible? Give a reason. If yes, find AB. [Year 2011]

Q2: If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ and I is the identity matrix of the same order and A^t is the transpose of matrix A, find $A^tB + BI$. [Year 2011]

Q3: If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find $A^2 - 5A + 7I$. [Year 2012]

Q4: Given $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$, write: (i) the order of the matrix X (ii) the matrix X. [Year 2012]

Q5: Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$, find the matrix X such that $A + 2X = 2B + C$ [Year 2013]

Q6: Find x and y if $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$ [Year 2013]

Q7: Find x, y if $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} y \\ 3 \end{bmatrix}$ [Year 2014]

Q8: Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$. Find $A^2 + AC - 5B$ [Year 2014]

Q9: If $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$ find x and y when $A^2 = B$ [Year 2015]

Q10: If $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$. Find $AB - 5C$. [Year 2015]

Q11: Given $A = \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = 9A + mI$. Find m. [Year 2016]

Q12: Given matrix $A = \begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

If $AX = B$, (i) write the order of matrix X
(ii) find the matrix X. [Year 2016]

Q13: If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ and $A^2 - 5B^2 = 5C$. Find matrix C where C is a 2 by 2 matrix. [Year 2017]

Matrices

Ms. Reena

Q14: Given matrix $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$. Find the matrix X if, $X = B^2 - 4B$

Hence, solve for 'a' and 'b', given

$X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

[Year 2017]

Q15: Find the value of 'x' and 'y' if

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

[Year 2018]

Q16: If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, find $AC + B^2 - 10C$.

[Year 2018]

Q17: Given $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6I$, where M is a matrix and I is unit matrix of order 2×2

(i) State the order of matrix M

(ii) Find the matrix M

[Year 2019]

Q18: Simplify $\sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix}$

$$+ \cos A \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

[Year 2019]

Q19: If $A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$, find

$A^2 - 2AB + B^2$

[Year 2020]