Tender Heart High School,Sector 33B,Chd.

Class : 10th   Subject: Mathematics   Date: 22.4.2024

Topic : Chapter- (Ratio and Proportion)

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→ In this chapter the following topics will be covered:-

I Proportion, Continued proportion, mean proportion

II Componendo, dividendo, alternendo, invertendo properties

III Direct simple applications on proportions

Ratio: Ratios are the mathematical numbers used to compare two things which are similar to each other in terms of units. e.g. ratio of ₹ 8 to ₹ 13 is 8 to 13

Using a colon → 8 : 13 or Using a fraction bar → $\frac{8}{13}$

Here, 8 is called the first term (antecedent) and 13 is called the second term (consequent)

Some facts about ratio :-

1. Ratio $\frac{a}{b}$ has no unit and can be written as $a:b$ (read as 'a' is to 'b')
2. The second term of a ratio cannot be zero.
3. A ratio must always be expressed in its lowest term e.g. 10 : 15 = 2 : 3
4. Ratio is taken only between positive quantities
5. A ratio is a number, so it has no units.
6. The order of the terms in a ratio is important e.g. $3:4 \neq 4:3$

Composition of ratio :-

1. Compound ratio : When two or more ratios are multiplied together. e.g. (i) Compound ratio of $a:b$ and $c:d$ is $ac:bd$
   (ii) Compound ratio of $a:b$, $c:d$ and $e:f$ is $ace:bdf$
2. Duplicate ratio of $a:b$ is $a^2:b^2$ e.g. 3:5 is 9:25
3. Triplicate ratio of $a:b$ is $a^3:b^3$ e.g. 3:5 is 27:125
4. Sub-duplicate ratio of $a:b$ is $\sqrt{a}:\sqrt{b}$ e.g. 3:5 is $\sqrt{3}:\sqrt{5}$
5. Sub-triplicate ratio of $a:b$ is $\sqrt[3]{a}:\sqrt[3]{b}$ e.g. 3:5 is $\sqrt[3]{3}:\sqrt[3]{5}$
6. Reciprocal ratio of $a:b$ is $b:a$ e.g. 3:5 is 5:3

Examples: (i) the duplicate ratio of 3:7 is $3^2:7^2 = 9:49$

(ii) the triplicate ratio of 2:5 is $2^3:5^3 = 8:125$

(iii) the sub duplicate ratio of 36:25 is $\sqrt{36}:\sqrt{25} = 6:5$

(iv) the sub triplicate ratio of 216:125 is $\sqrt[3]{216}:\sqrt[3]{125} = 6:5$

Proportion: An equality of two ratios.
Four (non-zero) quantities a, b, c, d are said to be in proportion iff $a:b = c:d$ or $a:b::c:d$ or $\frac{a}{b} = \frac{c}{d}$
It is read as "a is to b as c is to d"
Here, a = first term, b = second term, c = third term and d = fourth term.
Also, 'a' and 'd' are called extremes (end terms) and 'b' and 'c' are called means (middle terms)

Product of extremes = Product of means
i.e. $ad = bc$

Continued proportion: The (non-zero) quantities of the same kind a, b, c, d, e, f, ... are in continued proportion iff $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} = \dots$
Thus, a, b and c are in continued proportion if $a:b = b:c$
Example: 2, 4 and 8 are in continued proportion, $\frac{2}{4} = \frac{4}{8}$

Mean proportional: If a, b and c are in continued proportion, then b is called the mean proportional of 'a' and 'c'. Thus, $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac \Rightarrow b = \sqrt{ac}$

Example 1: Find the fourth proportional to 3, 6 and 4.5
Solution: Let the fourth proportional be x
Then, $3:6 = 4.5:x \Rightarrow 3 \times x = 6 \times 4.5 \Rightarrow x = 9$

Example 2: Find the mean proportional between 6.25 and 0.16
Solution: Let the mean proportional be x
Then, 6.25, x and 0.16 are in continued proportion.

$$\Rightarrow 6.25 : x = x : 0.16 \Rightarrow x \times x = 6.25 \times 0.16$$
$$\Rightarrow x^2 = \frac{625}{100} \times \frac{16}{100} = 1 \Rightarrow x = \sqrt{1} = 1$$

Example 3: Find the third proportional to 1.2 and 1.8
Solution: Let the third proportional be x
Then, 1.2, 1.8 and x are in continued proportion.

$$\Rightarrow 1.2 : 1.8 = 1.8 : x \Rightarrow x = \frac{1.8 \times 1.8}{1.2} = 2.7$$

Note: If a, b, c are in continued proportion, then
a → first proportional, b → mean proportional
c → third proportional

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Properties of Proportion :-

1. Invertendo :- If $a:b = c:d$, then $b:a = d:c$
   Example :- If $3:4 = 5:7$, then $4:3 = 7:5$
2. Alternendo :- If $a:b = c:d$, then $a:c = b:d$
   Example :- If $3:4 = 5:7$, then $3:5 = 4:7$
3. Componendo :- If $a:b = c:d$, then $a+b : b = c+d : d$
   Example :- If $3:4 = 5:7$, then $3+4 : 4 = 5+7 : 7$
4. Dividendo :- If $a:b = c:d$, then $a-b : b = c-d : d$
   Example :- If $a:b = c:d$, then $3-4 : 4 = 5-7 : 7$
5. Componendo and Dividendo :-
   If $a:b = c:d$, then $a+b : a-b = c+d : c-d$

Note : Proof of above properties are given in book at page no. (6.7)

6. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then each ratio $= \frac{a+b+c}{b+d+f} = \frac{\text{sum of antecedents}}{\text{sum of consequents}}$

Example 1: If $x : y = 2 : 3$, find the value of $3x + 2y : 2x + 5y$

Solution :- 1st Method

$$3x + 2y : 2x + 5y = \frac{3x+2y}{2x+5y} = \frac{3\left(\frac{x}{y}\right)+2}{2\left(\frac{x}{y}\right)+5} \quad \left[\text{Dividing each term by } 'y'\right]$$

$$= \frac{3 \times \frac{2}{3} + 2}{2 \times \frac{2}{3} + 5} \quad \left[\because \frac{x}{y} = \frac{2}{3}\right] \quad = 12 : 19 \quad \text{Answer}$$

2nd Method

Given $x : y = 2 : 3 \Rightarrow 3x = 2y \Rightarrow x = \frac{2y}{3}$ or $y = \frac{3x}{2}$

Then, $$\frac{3x+2y}{2x+5y} = \frac{3 \times \frac{2y}{3} + 2y}{2 \times \frac{2y}{3} + 5y} = \frac{4y}{\frac{19}{3}y} = \frac{4 \times 3}{19} = 12 : 19$$

3rd Method

Given $x : y = 2 : 3 \Rightarrow$ Let $x = 2k$ and $y = 3k$

Then, $$\frac{3x+2y}{2x+5y} = \frac{3 \times 2k + 2 \times 3k}{2 \times 2k + 5 \times 3k} = \frac{6k+6k}{4k+15k} = \frac{12k}{19k} = 12 : 19$$

Important Note :-

For $x : y = 2 : 3$ if we take $x = 2$ and $y = 3$

then $\frac{3x+2y}{2x+5y} = \frac{3 \times 2 + 2 \times 3}{2 \times 2 + 5 \times 3} = \frac{12}{19} = 12 : 19$, which is same as obtained in each solution given above. But this solution is absolutely wrong and for this solution, a student will score no marks.

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Example 2:- If $p = \frac{4xy}{x+y}$, find the value of $\frac{p+2x}{p-2x} + \frac{p+2y}{p-2y}$.

Solution:- 1st Method

Given, $p = \frac{4xy}{x+y} \Rightarrow \frac{p}{2x} = \frac{2y}{x+y}$ Now, applying componendo and dividendo, we get

$$\frac{p+2x}{p-2x} = \frac{2y+x+y}{2y-x-y} = \frac{x+3y}{y-x} \quad \text{--------- (i)}$$

Again, $p = \frac{4xy}{x+y} \Rightarrow \frac{p}{2y} = \frac{2x}{x+y}$ Now, applying componendo and dividendo, we get

$$\frac{p+2y}{p-2y} = \frac{2x+x+y}{2x-x-y} = \frac{3x+y}{x-y} \quad \text{------ (ii)}$$

Now, adding equation (i) and (ii) on both sides, we get

$$\frac{p+2x}{p-2x} + \frac{p+2y}{p-2y} = \frac{x+3y}{y-x} + \frac{3x+y}{x-y}$$

$$= \frac{x+3y}{y-x} - \frac{3x+y}{y-x} = \frac{x+3y-3x-y}{y-x} = 2$$

2nd Method

Given, $p = \frac{4xy}{x+y} \Rightarrow \frac{p+2x}{p-2x} + \frac{p+2y}{p-2y} = \frac{\frac{4xy}{x+y}+2x}{\frac{4xy}{x+y}-2x} + \frac{\frac{4xy}{x+y}+2y}{\frac{4xy}{x+y}-2x}$

$$= \frac{4xy+2x(x+y)}{4xy-2x(x+y)} + \frac{4xy+2y(x+y)}{4xy-2y\;x+y}$$

$$= \frac{4xy+2x^2+2xy}{4xy-2x^2-2xy} + \frac{4xy+2xy+2y^2}{4xy-2xy-2y^2} = \frac{6xy+2x^2}{2xy-2x^2} + \frac{6xy+2y^2}{2xy-2y^2}$$

$$= \frac{2x(3y+x)}{2x(y-x)} + \frac{2y(3x+y)}{2y(x-y)} = \frac{3y+x}{y-x} - \frac{3x+y}{y-x}$$

$$= \frac{3y+x-3x-y}{y-x} = \frac{2y-2x}{y-x} = 2$$

$$\left[\because \frac{3x+y}{x-y} = -\frac{3x+y}{y-x}\right]$$

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K - Method

Case I :- If a, b, c, d are in proportion then $\frac{a}{b} = \frac{c}{d} = k$ (say)

$\Rightarrow a = bk$ and $c = dk$

Example 1 - If a, b, c, d are in proportion, prove that $a+b : c+d = \sqrt{2a^2+7b^2} : \sqrt{2c^2+7d^2}$

Let $\frac{a}{b} = \frac{c}{d} = k \Rightarrow a = bk$ and $c = dk$

$$L.H.S. = \frac{a+b}{c+d} = \frac{kb+b}{kd+d} = \frac{b(k+1)}{d(k+1)} = \frac{b}{d}$$

$$R.H.S. = \frac{\sqrt{2a^2+7b^2}}{\sqrt{2c^2+7d^2}} = \frac{\sqrt{2k^2b^2+7b^2}}{\sqrt{2k^2d^2+7d^2}} = \frac{\sqrt{b^2(2k^2+7)}}{\sqrt{d^2(2k^2+7)}}$$

$$= \sqrt{\frac{b^2}{d^2}} = \frac{b}{d}$$ Hence, L.H.S = R.H.S.

Case II :- If a, b, c, d are in continued proportion then, $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$ (say)

$\Rightarrow c = dk,\ b = ck = kd.k = k^2d$
and $a = bk = k^2d.k = k^3d$

Example 2 - If a, b, c, d are in continued proportion, prove that $\sqrt{ab} + \sqrt{bc} - \sqrt{bd} = \sqrt{(a+b-c)(b+c-d)}$

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k \Rightarrow a = k^3d,\ b = k^2d,\ c = dk$

$$L.H.S. = \sqrt{ab} + \sqrt{bc} - \sqrt{bd} = \sqrt{k^3d.k^2d} + \sqrt{k^2d.kd} - \sqrt{kd.d}$$
$$= \sqrt{k^5d^2} + \sqrt{k^3d^2} - \sqrt{kd^2} = d\sqrt{k}(k^2+k-1)$$

$$R.H.S. = \sqrt{(a+b-c)(b+c-d)}$$
$$= \sqrt{(k^3d+k^2d-kd)(k^2d+kd-d)}$$
$$= \sqrt{kd(k^2+k-1).d(k^2+k-1)}$$
$$= d\sqrt{k}(k^2+k-1)$$

Hence, L.H.S = R.H.S.

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Example 3:- If a, b, c are in continued proportion, prove that $a:c :: (3a^2+5ab+7b^2):(3b^2+5bc+7c^2)$

Solution:- Let $\frac{a}{b} = \frac{b}{c} = k \Rightarrow a = bk, b = ck$

$\Rightarrow a = ck.k = ck^2$ so, $a = ck^2$ and $b = ck$

$L.H.S. = \frac{a}{c} = \frac{ck^2}{c} = k^2$

$$R.H.S. = \frac{3a^2+5ab+7b^2}{3b^2+5bc+7c^2} = \frac{3(ck^2)^2+5(ck^2).ck+7(ck)^2}{3(ck)^2+5(ck)c+7c^2}$$

$$= \frac{3c^2k^4+5c^2k^3+7c^2k^2}{3c^2k^2+5c^2k+7c^2} = \frac{c^2k^2(3k^2+5k+7)}{c^2(3k^2+5k+7)}$$

$= k^2$ Hence, L.H.S = R.H.S.

Direct Simple Applications of Proportion :-

Example 4:- Divide ₹ 3740 into three parts in such a way that half of the first part, one-third of the second part and one-sixth of the third part are equal.

Solution:- Let $\frac{1}{2}$ (1st part) $= \frac{1}{3}$ (2nd part) $= \frac{1}{6}$ (3rd part) $= x$

$\Rightarrow$ 1st part $= 2x$, 2nd part $= 3x$, 3rd part $= 6x$

According to given, $2x + 3x + 6x = 3740$

$\Rightarrow 11x = 3740 \Rightarrow x = 340$

Therefore, 1st part $= 2 \times 340 = 680$, 2nd part $= 3 \times 340 = 1020$ and 3rd part $= 6 \times 340 = 2040$

Example 5:- An employer reduces the number of employees in the ratio of 10:7 and increases their wages in the ratio of 14:15. In what ratio, the wage bill is increased or decreased?

Solution:- Let the number of employees be $10x$ and wages per head be ₹$14y$

So, total wage bill $=$ ₹ $10x \times 14y =$ ₹ $140xy$

At present, the number of employee $= 7x$

and wages per head = ₹ $15y$
so, total wage bill = ₹ $(7x \times 15y)$ = ₹ $105xy$
Ratio in which the wage bill is decreased
$= 140xy : 105xy = 4:3$
Hence, the wage bill is decreased in the ratio 4:3

Example 6:- In a mixture of 63 litres, the ratio of milk and water is 5:2. How much water must be added to this mixture to make the ratio 3:2?

Solution:- Given, milk : water = 5 : 2 [sum = 7]
Quantity of milk in it $= 63 \times \frac{5}{7} = 45$ litres

Quantity of water in it $= 63 - 45 = 18$ litres
Let the quantity of water to be added be x litres
Then quantity of milk in new mixture = 45 litres
And quantity of water in new mixture $= 18 + x$

$\Rightarrow \frac{45}{18+x} = \frac{3}{2} \Rightarrow 54 + 3x = 90 \Rightarrow 3x = 36$

$\Rightarrow x = 12$ Therefore 12 litres to be added.

Types	Formulae
Invertendo: If a: b= c: d then	b:a :: d: c or $\frac{b}{a} = \frac{d}{c}$
Alterando: If a: b= c: d then	a: c :: b: d or $\frac{a}{c} = \frac{b}{d}$
Componendo: If a: b= c: d then	$\frac{a+b}{b} = \frac{c+d}{d}$
Dividendo: If a: b= c: d then	$\frac{a-b}{b} = \frac{c-d}{d}$
Componendo & Dividendo: If a: b= c: d then	$\frac{a+b}{a-b} = \frac{c+d}{c-d}$

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