Tender Heart High School,Sector 33B,Chd.

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Topic: Chapter-5 Quadratic Equation (contd..)

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Nature of the roots of a Quadratic Equation

We know that, in quadratic equation $ax^2+bx+c=0$, roots are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

The nature of the roots depends on the quantity under the square root sign i.e. b^2-4ac

This is called discriminant of the equation and is denoted by 'D'

Therefore, Discriminant $D = b^2-4ac$

Now, the following three cases arises :-

Case I : If b^2-4ac is positive i.e. $b^2-4ac>0$, then the roots are real and distinct.

(i) If b^2-4ac is a perfect square, the roots are real, rational and distinct (unequal).

(ii) If b^2-4ac is not a perfect square, the roots are real, irrational and distinct.

Examples :-

1) $2x^2+x-3=0$, Here $a=2, b=1, c=-3$

$D = b^2-4ac = (1)^2-4(2)(-3) = 25 = (5)^2 > 0$

Since, D is positive and perfect square, therefore roots are real, rational and distinct.

2) $2x^2+2x-3=0$, Here $a=2, b=2, c=-3$

$D = b^2-4ac = (2)^2-4(2)(-3) = 28 > 0$ is not a perfect square, therefore roots are irrational and distinct.

Case II : If $b^2-4ac=0$, the roots are real and equal

Example :- $4x^2+4x+1=0$ Here, $a=4, b=4, c=1$

$D = b^2-4ac = (4)^2-4(4)(1) = 16-16 = 0$

Since, $D=0$, therefore, roots are real and equal.

Case III : If b^2-4ac is negative, i.e. $b^2-4ac<0$, then, the roots are not real (imaginary).

Example : $x^2-2x+5=0$ Here $a=1, b=-2, c=5$

$$D = b^2-4ac = (-2)^2-4(1)(5) = 4-20 = -16<0$$

Since, D is negative, therefore, the given equation does not have real roots.

So, we conclude that

$b^2-4ac=0$, roots are real and equal

$b^2-4ac>0$, roots are real and unequal (distinct)

$b^2-4ac\geq 0$, roots are real (rational or irrational)

$b^2-4ac<0$, roots are imaginary (not real)

Example 1: Find the values of 'k' for which the equation $x^2-4x+k=0$ has distinct real roots.

Solution : The given equation is $x^2-4x+k=0$

Here, $a=1$, $b=-4$, $c=$ 'k'

Now, $D = b^2-4ac = (-4)^2-4\times 1\times k = 16-4k$

For real and distinct roots, $D>0$ i.e. $16-4k>0 \Rightarrow 16>4k$

$\Rightarrow 4k<16 \Rightarrow k<\frac{16}{4} \Rightarrow k<4$

Therefore, the given equation will have distinct real roots if $k<4$.

Example 2 : If -4 is a root of the quadratic equation $x^2+px-4=0$ and the quadratic equation $x^2+px+k=0$ has equal roots, find the value of 'k'.

Solution : Since -4 is a root of the equation $x^2+px-4=0$

$\Rightarrow D=b^2-4ac=0 \Rightarrow (-4)^2+p(-4)-4=0 \Rightarrow 16-4p-4=0$

$\Rightarrow 12-4p=0 \Rightarrow 4p=12 \Rightarrow p=3$

Putting $p=3$ in the equation $x^2+px+k=0$, the equation becomes $x^2+3x+k=0$ Here, $a=1, b=3, c=k$

$\Rightarrow D = b^2-4ac = 9-4\times 1\times k = 9-4k$

For equal roots $D=0$ i.e. $9-4k=0 \Rightarrow 9=4k \Rightarrow k=\frac{9}{4}$

NOTE : In case of $D=0$, we have two equal roots i.e. one real roots (repeated twice)

PROBLEMS ON QUADRATIC EQUATIONS :-

1) Read the statement of the problem carefully atleast twice, and determine what quantity must be found.
2) Represent the unknown quantity by a variable.
3) Form the quadratic equation with the help of the given condition or statement
4) Solve the resulting equation.

NOTE : 1) Lengths, areas, volume, money etc is always taken as positive, neglecting the negative values.
2) Check the answer obtained by determining whether or not they fulfill the condition of the original problem.

Example 1: The sum of two natural number is 8. Determine the numbers, if the sum of their reciprocals is $\frac{8}{15}$.

Solution : As the sum of two natural numbers is 8, let the numbers be x and $8-x$ where $x<8$

According to given, $\frac{1}{x}+\frac{1}{8-x}=\frac{8}{15}$

$\Rightarrow \frac{8-x+x}{x(8-x)}=\frac{8}{15} \Rightarrow \frac{8}{x(8-x)}=\frac{8}{15} \Rightarrow 15=x(8-x)$

$\Rightarrow 15=8x-x^2 \Rightarrow x^2-8x+15=0 \Rightarrow x^2-3x-5x+15=0$

$\Rightarrow x(x-3)-5(x-3)=0 \Rightarrow (x-3)(x-5)=0 \Rightarrow x-3=0$ or $x-5=0$

$\Rightarrow x=3$ or $x=5$

When $x=3$, then the other number is $8-3=5$

When $x=5$, then the other number is $8-5=3$

Hence, the required numbers are 3 and 5.

NOTE : 1) We can verify the answer by putting values in the equation given.
2) In the above example if difference of two natural number is 8, the we assume numbers be x and $x+8$.
3) Since $x+8>x \Rightarrow \frac{1}{x+8}<\frac{1}{x}$

Example 2: A two digit number is such that the product of its digits is 12. When 36 is added to this number, the digits interchange their places. Find the number.

Solution: Let the unit's digit of the two digit number be x.

Since the product of its digits is 12, its ten's digit $= \frac{12}{x}$

$\therefore$ The number $= 10 \times \frac{12}{x} + x$

On interchanging the digits, the number $= 10 \times x + \frac{12}{x}$

According to given, $10 \cdot x + \frac{12}{x} = \left(10 \times \frac{12}{x} + x\right) + 36$

$\Rightarrow 10x + \frac{12}{x} = \frac{120}{x} + x + 36 \Rightarrow 10x^2 + 12 = 120 + x^2 + 36x$

$\Rightarrow 9x^2 - 36x - 108 = 0 \Rightarrow x^2 - 4x - 12 = 0 \Rightarrow x^2 - 6x + 2x - 12 = 0$

$\Rightarrow x(x-6) + 2(x-6) = 0 \Rightarrow (x-6)(x+2) = 0$

$\Rightarrow x = 6$ or $x = -2$ (but x being a digit of a number cannot be negative)

$\Rightarrow x = 6$

Therefore, Unit's digit $= 6$ and ten's digit $= \frac{12}{6} = 2$

Hence, the required number is 26.

Note: Consecutive natural numbers are $x, x+1, x+2, \ldots$

same [Consecutive odd integers are $x, x+2, x+4, \ldots$
Consecutive even integers are $x, x+2, x+4, \ldots$

Don't Forget...

- Quadratic Equations generally yield <u>two</u> answers.
- Length can never be negative.
- So only use the <u>positive</u> answers in an area problem.

REVISION

Problems on Quadratic Equations

Ms. Reena

Q1 ₹480 is divided equally among 'x' children. If the number of children were 20 more, than each would have got ₹12 less. Find 'x'

[year 2011]

Q2 A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/h more, the time taken for the journey would have been 1 hour 40 minutes less. Find the original speed of the car. [year 2012]

Q3 A shopkeeper purchases a certain number of books for ₹960. If the cost per book was ₹8 less, the number of books that could be purchased for ₹960 would be 4 more. Write an equation, taking the original cost of each book to be ₹ x, and solve it to find the original cost of the books. [year 2013]

Q4 A 2-digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number. [year 2014]

Q5 Sum of two natural numbers is 8 and the difference of their reciprocals is $\frac{2}{15}$. Find the numbers.

[year 2015]

Q6 A bus covers a distance of 240 km at a uniform speed. Due to heavy rain its speed gets reduced by 10 km/hr and as such it takes two hours longer to cover the total distance. Assuming the uniform speed to be 'x' km/h, form an equation and solve it to evaluate 'x'

[year 2016]

Q7 The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages.
[year 2017]

Q8 ₹ 7,500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹100 more. Find the original number of children.
[year 2018]

Q9 The product of two consecutive natural numbers which are multiples of 3 is equal to 810. Find the two numbers. [year 2019]

Q10 The difference of two natural numbers is 7 and their product is 450. Find the numbers.
[year 2020]