Class 10, Mathematics Date: 1.10.2024

Chapter- 22 Trigonometric Identities Ms. Reena

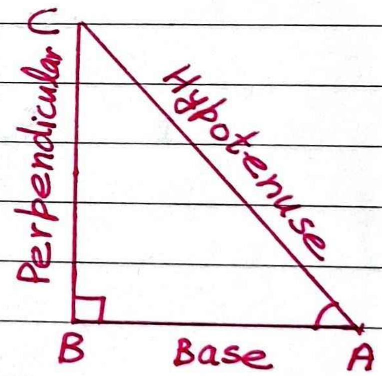
Introduction:- An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle(s) involved

Trigonometry means; "the science which deals with the measurements of triangles.

Trigonometrical Ratios:-

There are six trigonometrical ratios relating to the three sides of a right-angled triangle (this has already been done by students in Class IX)

For an acute angle of a right-angled triangle:-



1) sine (sin) = $\frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$\Rightarrow \sin A = \frac{BC}{AC}$ and $\sin C = \frac{AB}{AC}$

2) cosine (cos) = $\frac{\text{Base}}{\text{Hypotenuse}}$

$\Rightarrow \cos A = \frac{AB}{AC}$ and $\cos B = \frac{BC}{AC}$

3) tangent (tan) = $\frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow \tan A = \frac{BC}{AB}$ and $\tan C = \frac{AB}{BC}$

4) cotangent (cot) = $\frac{\text{Base}}{\text{Perpendicular}}$

$\Rightarrow \cot A = \frac{AB}{BC}$ and $\cot B = \frac{BC}{AB}$

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5) secant (sec) $= \frac{\text{Hypotenuse}}{\text{Base}}$

$\Rightarrow \sec A = \frac{AC}{AB}$ and $\sec B = \frac{AC}{BC}$

6) cosecant (cosec) $= \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

$\Rightarrow \operatorname{cosec} A = \frac{AC}{BC}$ and $\operatorname{cosec} B = \frac{AC}{AB}$

Note :- Each trigonometrical ratio is a real number and has no unit.

Reciprocal relations:-

1) $\sin A = \frac{1}{\operatorname{cosec} A} \Rightarrow \sin A . \operatorname{cosec} A = 1$

2) $\cos A = \frac{1}{\sec A} \Rightarrow \cos A . \sec A = 1$

3) $\tan A = \frac{1}{\cot A} \Rightarrow \tan A . \cot A = 1$

Quotient relations:-

$\frac{\sin A}{\cos A} = \tan A$ and $\frac{\cos A}{\sin A} = \cot A$

Square relations:-

1) $\sin^2 A + \cos^2 A = 1$

$\Rightarrow \sin^2 A = 1 - \cos^2 A$ and $\cos^2 A = 1 - \sin^2 A$

$\Rightarrow \sin^2 A = (1 + \cos A)(1 - \cos A)$

and $\cos^2 A = (1 + \sin A)(1 - \sin A)$

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2) $1 + \tan^2 A = \sec^2 A$

$\Rightarrow \sec^2 A - \tan^2 A = 1$ and $\sec^2 A - 1 = \tan^2 A$

3) $1 + \cot^2 A = \text{cosec}^2 A$

$\Rightarrow \text{cosec}^2 A - \cot^2 A = 1$ and $\text{cosec}^2 A - 1 = \cot^2 A$

Important :-

In order to prove a trigonometrical identity :-

→ Start with any side lef-hand side i.e. L.H.S. or right-hand side (R.H.S) and by applying trigonometrical relations reach to the other side.

i.e. if you start with L.H.S. then reach to R.H.S.
if you start with R.H.S. then reach to L.H.S.

→ In general, start with the more complicated side.

Example 1:- Prove that $\tan A + \cot A = \sec A . \text{cosec} A$

Solution :- L.H.S. $= \tan A + \cot A$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{1}{\cos A \sin A}$$

$$= \sec A . \text{cosec} A$$

$$= \text{R.H.S.}$$

$$\left[\because \sec A = \frac{1}{\cos A} \text{ and } \text{cosec} A = \frac{1}{\sin A}\right]$$

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Example 2:- $\cos^4 A - \sin^4 A = 2\cos^2 A - 1$

$$\begin{aligned}
\text{L.H.S.} &= \cos^4 A - \sin^4 A \\
&= (\cos^2 A)^2 - (\sin^2 A)^2 \\
&= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A) \\
&= \cos^2 A - \sin^2 A \qquad [\because \cos^2 A + \sin^2 A = 1] \\
&= \cos^2 A - (1 - \cos^2 A) \\
&= 2\cos^2 A - 1 \qquad [\because \sin^2 A = 1 - \cos^2 A] \\
&= \text{R.H.S.}
\end{aligned}$$

Example 3:- $\dfrac{\sin A}{1+\cos A} + \dfrac{1+\cos A}{\sin A} = 2\operatorname{cosec} A$

$$\begin{aligned}
\text{L.H.S.} &= \frac{\sin^2 A + (1+\cos A)^2}{(1+\cos A)\sin A} \\
&= \frac{\sin^2 A + 1 + \cos^2 A + 2\cos A}{(1+\cos A)\sin A} \\
&= \frac{1+1+2\cos A}{(1+\cos A)\sin A} \qquad [\because \sin^2 A + \cos^2 A = 1] \\
&= \frac{2(1+\cos A)}{(1+\cos A)\sin A} = \frac{2}{\sin A} = 2\operatorname{cosec} A = \text{R.H.S.}
\end{aligned}$$

Example 4:- $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

$$\begin{aligned}
\text{L.H.S.} &= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)} \\
&= \frac{\sin A[1 - 2(1-\cos^2 A)]}{\cos A(2\cos^2 A - 1)} \\
&= \frac{\sin A(2\cos^2 A - 1)}{\cos A(2\cos^2 A - 1)} \\
&= \frac{\sin A}{\cos A} = \tan A = \text{R.H.S.}
\end{aligned}$$

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Trigonometric Tables

→ The tables of natural sines, cosines, tangents, etc, given at the end of the book [page(v) to page(xvi)] give the values of these T-ratios upto 4-decimal places, and are known as trigonometric tables.

A Trigonometric Table consists of three parts:-

(i) A column on the extreme left containing degree from $0°$ to $89°$

(ii) Ten columns headed by $0'$, $6'$, $12'$, $18'$, $24'$, $30'$, $36'$, $42'$, $48'$ and $54'$

(iii) Five columns of mean differences, headed by $1'$, $2'$, $3'$, $4'$ and $5'$

Note:-

(i) The mean difference is added in case of sines, tangents and secants

(ii) The mean difference is subtrated in case of cosines, cotangents and cosecants

Relation Between Degrees and Minutes

$$1° = 60' \Rightarrow 1' = \left(\frac{1}{60}\right)^\circ$$

$$\Rightarrow 6' = \left(\frac{1}{60} \times 6\right)^\circ = \left(\frac{1}{10}\right)^\circ = (0.1)^\circ$$

Thus, $(40.1)°$ means $40°6'$

$(40.2)°$ means $40°12'$ and so on

Example 1 Find $\sin 36°51'$

$x°$	0'	6' 12' 18'	24' 30' 36'	42' 48' 54'
36	0.5878			5990

1' 2' 3' 4' 5' Difference to add
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since $\sin 36°51' = \sin(36°48' + 3')$

From table,

$\sin 36°48' = 0.5990$

[see the number in the row against 36° and in the column headed 48']

diff for 3' = 0.0007 (To add)

[see the number in the same row and under 3']

Therefore, $\sin 36°51' = 0.5997$

Example 2 Find $\tan 38°27'$

We have, $38°27' = 38°24' + 3'$

In the table of natural tangents, look at the number in the row against 38° and in the column headed 24', as shown below:-

From the Table of Natural Tangents

$x°$	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
38°	0.7813				0.7926					

1' 2' 3' 4' 5'
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$\tan 38°24' = 0.7926$

diff for 3' = 0.0014 (To add)

$\therefore \tan 38°27' = 0.7926 + 0.0014 = 0.7940$

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Example 3 Find $\cos 24°17'$

We have, $24°17' = 24°12' + 5'$

In the table of natural cosines, look at the number in the row against 24° and in the column headed by 12', as shown below:-

From table of natural cosines

$x°$	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
24°	0.9135		0.9121							

Mean Difference

1'	2'	3'	4'	5'
				6

$\therefore \cos 24°12' = 0.9121$

Mean Diff. for $5' = 0.0006$

(To be subtrated)

$\cos 24°17' = 0.9121 - 0.0006 = 0.9115$

Reverse use of Tables:-

Example 1 Find θ, when $\sin\theta = 0.7114$

Solution:- From the table, find the angle whose sine is just smaller than 0.7114

We have $\sin\theta = 0.7114$

$\sin 45°18' = 0.7108$

Difference $= 0.0006$

Mean difference 6 corresponding to 3'

Therefore, required angle is $(45°18' + 3')$

$= 45°21'$

Note:- No direct questions from chapter-18 will come in ICSE Board Exam.

This chapter is helful in solving questions of next chapter i.e. Ch-19.