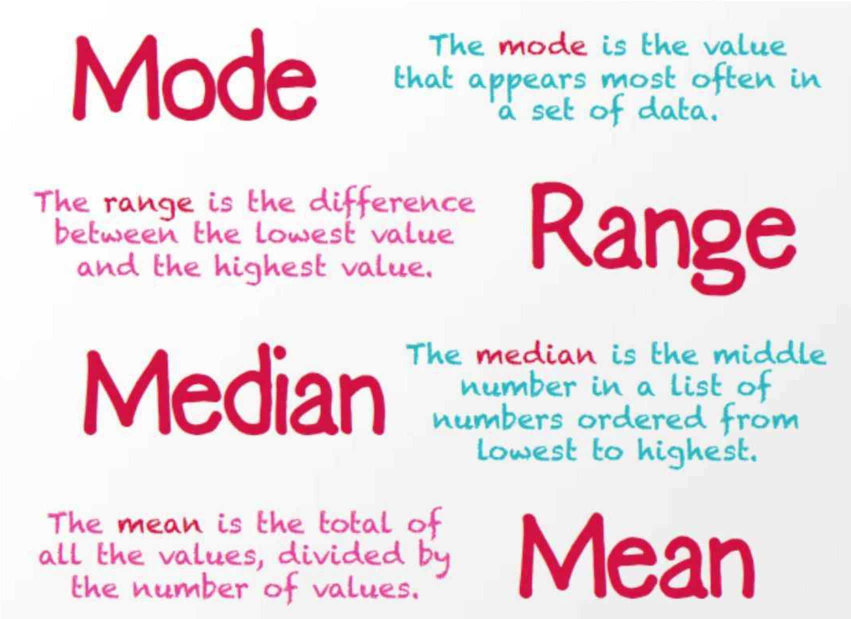
Class: 10th　　　　　　　　　　Date: 19.8.24

Subject: Mathematics　　　　　　Teacher: Ms. Reena

Chapter – 26
Measures of Central Tendency
Median, Quartiles and Mode



Students in the previous assignment we have discussed about mean and three different methods to find mean. In this assignment we will be discussing about the remaining two measures of Central Tendency that is Median and Mode.

MEDIAN

Median is the central value or middle observation of a statistical data if it is arranged in ascending or descending order. Thus, if there are 'n' observations or variates $x_1, x_2, x_3, ---, x_n$ arranged in ascending or decending order, then median is defined as the middle observation if the number of observations is odd and as the mean of the two middle observations if the number of observations is even. So, there will be an equal number of observations above and below the median.

$$\text{Median} = \begin{cases} \left(\frac{n+1}{2}\right)^{th} \text{ observation, if } n \text{ is odd} \\ \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{ observation} + \left(\frac{n}{2}+1\right)^{th} \text{ observation}\right], \\ \text{if } n \text{ is even} \end{cases}$$

So, median is that value of the given number of observations, which divides it into exactly two parts.

Let us consider an example :-

Consider the numbers as follows :-

15, 72, 44, 26, 66, 38, 91

Step 1 :- To arrange them in ascending order
15, 26, 38, 44, 66, 72, 91

Step 2 :- Check the number of terms that is odd or even.
Here, no. of terms is 7 which is odd

Step 3 :- Median $= \left(\frac{n+1}{2}\right)^{th}$ observation

$= \left(\frac{7+1}{2}\right)^{th} = 4^{th}$ observation

$= 44$

So, Median $= 44$, which is the middle most value.

Note :- If the number of observation is even then the median is the average of $\frac{n}{2}$ and $\frac{n}{2}+1$ term.

Now, let us discuss about median of Discrete Frequency Distribution that is Grouped Data.

When data are presented as a frequency distribution then we find the cumulative frequency (c.f.) as it is the simplest way to find the middle value.

Let us understand this with the help of an example.

Example:- The following table gives a record of marks obtained by 41 students of a class:-

Marks	20	15	17	30	22	25
No. of students	10	3	5	4	11	8

Arranging the given terms in an ascending order and preparing a cumulative frequency table, we have

Marks	Frequency	Cumulative frequency (c.f.)
15	3	3
17	5	8
20	10	18
22	11	29
25	8	37
30	4	41

Here, total number of students = 41

So, $n = 41$, which is odd.

$\therefore$ Median $= \left(\frac{n+1}{2}\right)^{th}$ term $= \left(\frac{41+1}{2}\right)^{th}$ term

= marks of 21st term or student.
Now, in cumulative frequency 21 is lying between 18 and 29 that is $18 < 21 < 29$
Here, the cumulative frequency just greater than 21 is 29 and the value that is marks corresponding to 29 is 22.
Thus, median marks = 22
Hope this concept of median is clear to everyone.
Now, children let us discuss about Quartiles.

QUARTILES

The observations which divide the whole set of observations into four equal parts, are known as quartiles.

Lower Quartile (or First Quartile)

If the variates are arranged in ascending order, then the observation lying midway between the lower extreme and the median is called the lower quartile, or First quartile, to be denoted by Q_1

Upper Quartile (or Third Quartile)

If the variates are arranged in ascending order, then the observation lying midway between the median and the upper extreme is called the upper quartile or Third quartile, to be denoted by Q_3

Middle Quartile

The middle quartile is the median, denoted by Q_2
So, median is also known as middle quartile

Formulae (For Quartiles)

For an ungrouped data containing 'n' observation we have

$$\text{Lower Quartile } (Q_1) = \begin{cases} \left(\frac{n}{4}\right)^{th} \text{ observation, when } n \text{ is even} \\ \left(\frac{n+1}{4}\right)^{th} \text{ observation, when } n \text{ is odd} \end{cases}$$

$$\text{Upper Quartile } (Q_3) = \begin{cases} \left(\frac{3n}{4}\right)^{th} \text{ observation, when } n \text{ is even} \\ \frac{3(n+1)}{4}^{th} \text{ observation, when } n \text{ is odd} \end{cases}$$

Range

The difference between the values of the biggest and the smallest observations is called the range.

Interquartile Range

The difference between the upper quartile and the lower quartile is called the inter-quartile range.

Thus, interquartile range $= Q_3 - Q_1$

Semi-interquartile Range

$\frac{1}{2}(Q_3 - Q_1)$ is called the semi-interquartile range.

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Now, students let us discuss about the last topic of this chapter that is "Mode".

Mode (or modal value)

The value around which there is the greatest concentration is called mode.
In the case of individual data, the mode is the variate which occurs most frequently. So, mode of a statistical data is the variate which has maximum frequency.

Example:- Find the mode for the following frequency distribution:-

Marks obtained	0	2	3	4	6	7	9	10
No. of students	3	5	12	18	21	8	2	1

Solution:- In the given distribution, the variate 6 has the maximum frequency.
Therefore, mode $= 6$

Students, In a grouped frequency distribution, it is not possible to determine the mode just by looking at the frequencies of the classes. Here, we can only locate a class, called modal class, with maximum frequency.

Note:-

1) A data may not have a mode. For example, the data 2, 5, 0, 7, 4, 6 has no mode because no number occurs more number of times than any other number.
2) A data may have more than one modes. For example, the data 2, 5, 0, 3, 5, 7, 6, 3, 8 has two modes 3 and 5 because each of 3 and 5 is repeated twice.

A data set can have no mode, one, or many.
None : 1, 2, 3, 4, 6, 8, 9
One mode (Unimodal) : 1, 2, 3, 3, 4, 5
Two mode (Bimodal) : 1, 1, 2, 3, 4, 4, 5
Three mode (Trimodal) : 1, 1, 2, 3, 3, 4, 5, 5
Multimodal : More than one

Estimation of Mode From Histogram

A histogram shows frequencies of values. In other words, how often a value appears in a data set. In a continuous frequency distribution, the mode can be estimated from the histogram of given frequency distribution.

Procedure :-

The following steps must be followed to find the mode graphically.

Step 1 :- Represent the given data in the form of a Histogram. The height of the rectangles in the histogram is marked by the frequencies of the class interval. Identify the highest rectangle. This corresponds to the modal class of the series.

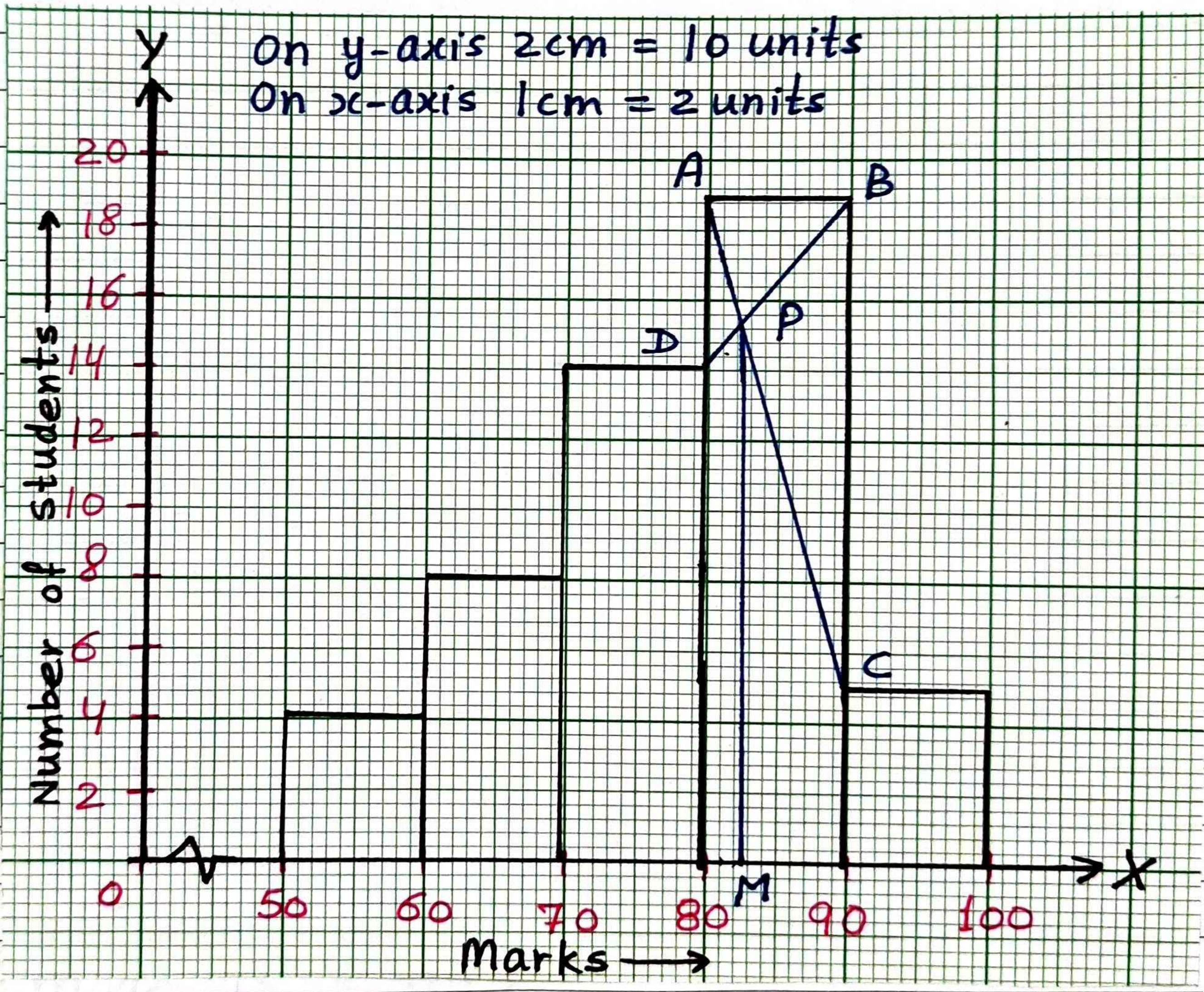
Step 2 :- Join the top corners of the modal rectangle with the immediately next corners of the adjacent rectangles. The two lines must be cutting each other.

Step 3 :- Through the point of intersection of the two straight lines drawn, draw a vertical line to meet the x-axis. (which is Mode).

Example 1:- A mathematics aptitude test of 50 students was recorded as follows:-

Marks	50-60	60-70	70-80	80-90	90-100
Number of students	4	8	14	19	5

Draw a histogram for the above data using a graph paper and locate the mode.



In this histogram, mark the upper corners of highest rectangle and the corners of adjoining rectangles as A, B, C, D as shown. Join AC and BD to intersect at point P. Draw PM ⊥ x-axis. Hence, Mode = 82.5

CUMULATIVE FREQUENCY CURVE (OGIVE)

Children you have done earlier the meaning of cumulative frequency and you are well aware that cumulative frequency means the added up frequencies when we go on adding the the frequencies then we get the cumulative frequencies Now, here we will learn about cumulative frequency curve also known as ogive.

An ogive graph is a plot used in statistics to show cumulative frequencies. It allows us to quickly estimate the number of observations that are less than or equal to a particular value.

Let us consider an example, and construct a cumulative frequency curve on graph.

Example 2:- Draw an ogive for the following frequency distribution.

Classes before adjustment	Classes after adjustment	Frequency	Cumulative Frequency
121 - 130	120.5 - 130.5	12	12
131 - 140	130.5 - 140.5	16	28
141 - 150	140.5 - 150.5	30	58
151 - 160	150.5 - 160.5	20	78
161 - 170	160.5 - 170.5	14	92
171 - 180	170.5 - 180.5	8	100

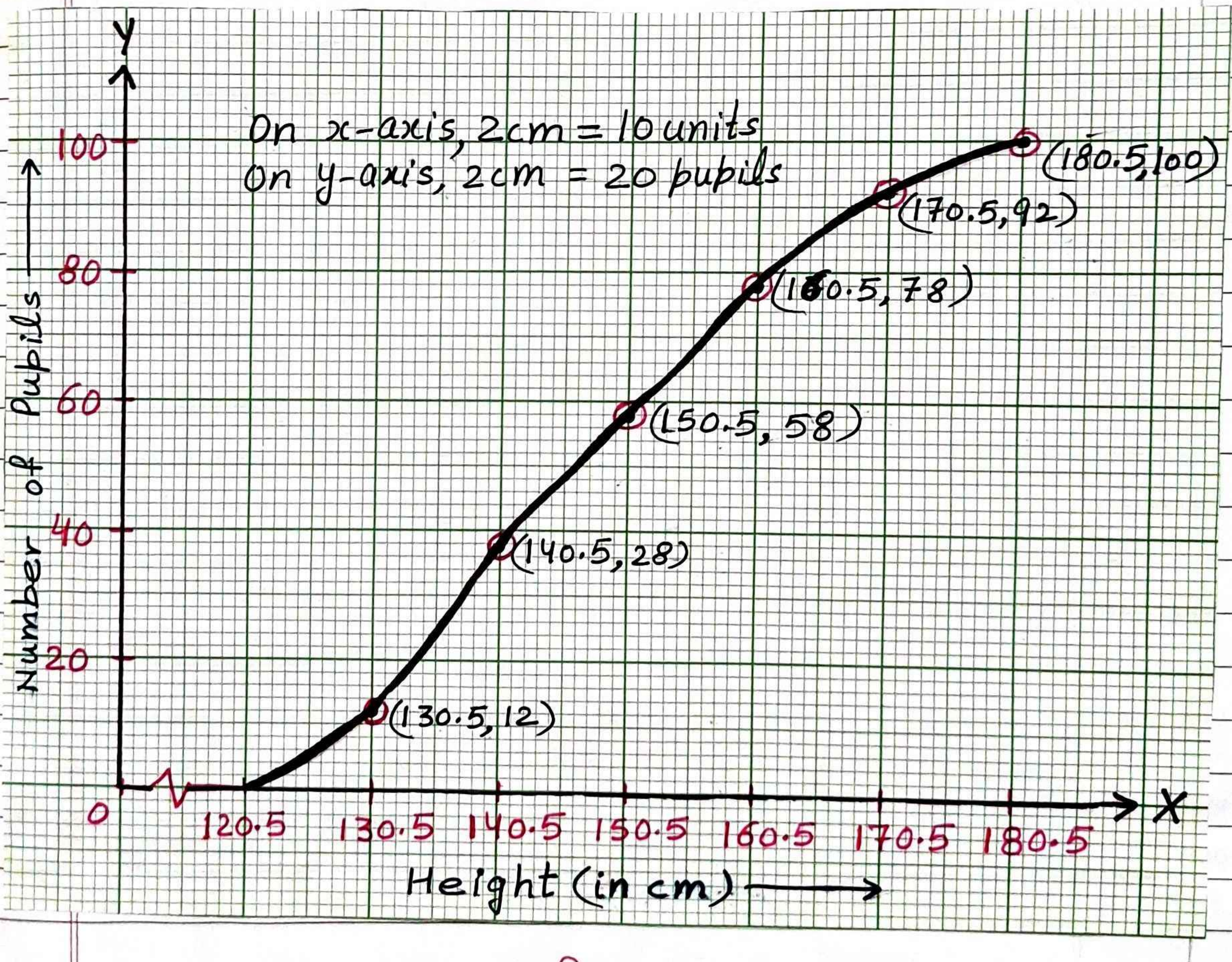
Step 1:- If the given frequency distribution is not continuous, convert it into the continuous form.

Step 2:- Prepare the cumulative frequency table.

Step 3:- Since the scale on x-axis starts at 120.5, a kink is shown near the origin on x-axis to indicate that the graph is drawn to scale beginning at 120.5

Step 4:- Plot the points representing upper class limits and the respective cumulative frequencies. Also plot the point representing lower limit of first class that is 120.5 - 130.5

Step 5:- Join these points by a freehand drawing. So, the required ogive is shown below:-



Students ogives can be used to find the median and quartiles of a frequency distribution. In order to determine the same, we may use the following steps:-

Step 1:- Draw cumulative frequency curve on the graph paper.

Step 2:- Compute $\frac{N}{2}$ and mark the corresponding point on y-axis.

where, N = total number of observations or sum of all frequencies

NOTE:- In graphical location, we consider median as $\frac{N}{2}$, whether N is odd or even.

Step 3:- Draw a line parallel to x-axis, from the point marked in step-2, cutting the cumulative frequency curve at a point P (say)

Draw perpendicular PM from P on the x-axis. The x- coordinate of point M gives the median.

Note:-

(i) To locate the value of Q_1 on ogive, we mark the point along y-axis, corresponding to $\frac{N}{4}$ and proceed similarly.

(ii) To locate the value of Q_3 on ogive, we mark the point along y-axis, corresponding to $\frac{3N}{4}$ and proceed similarly.

Now, let us discuss an example to find mean and quartiles with the help of cumulative frequency curve.

Example 3:- The following distribution represents the heights of 160 students of a school.

Height	140-145	145-150	150-155	155-160	160-165	165-170	170-175	175-180
Number of students	12	20	30	38	24	16	12	8

Draw an ogive for the given distribution taking 2cm = 5cm of height on one axis and 2cm = 20 students on the other axis
Using the graph, determine:-

(i) The Median height

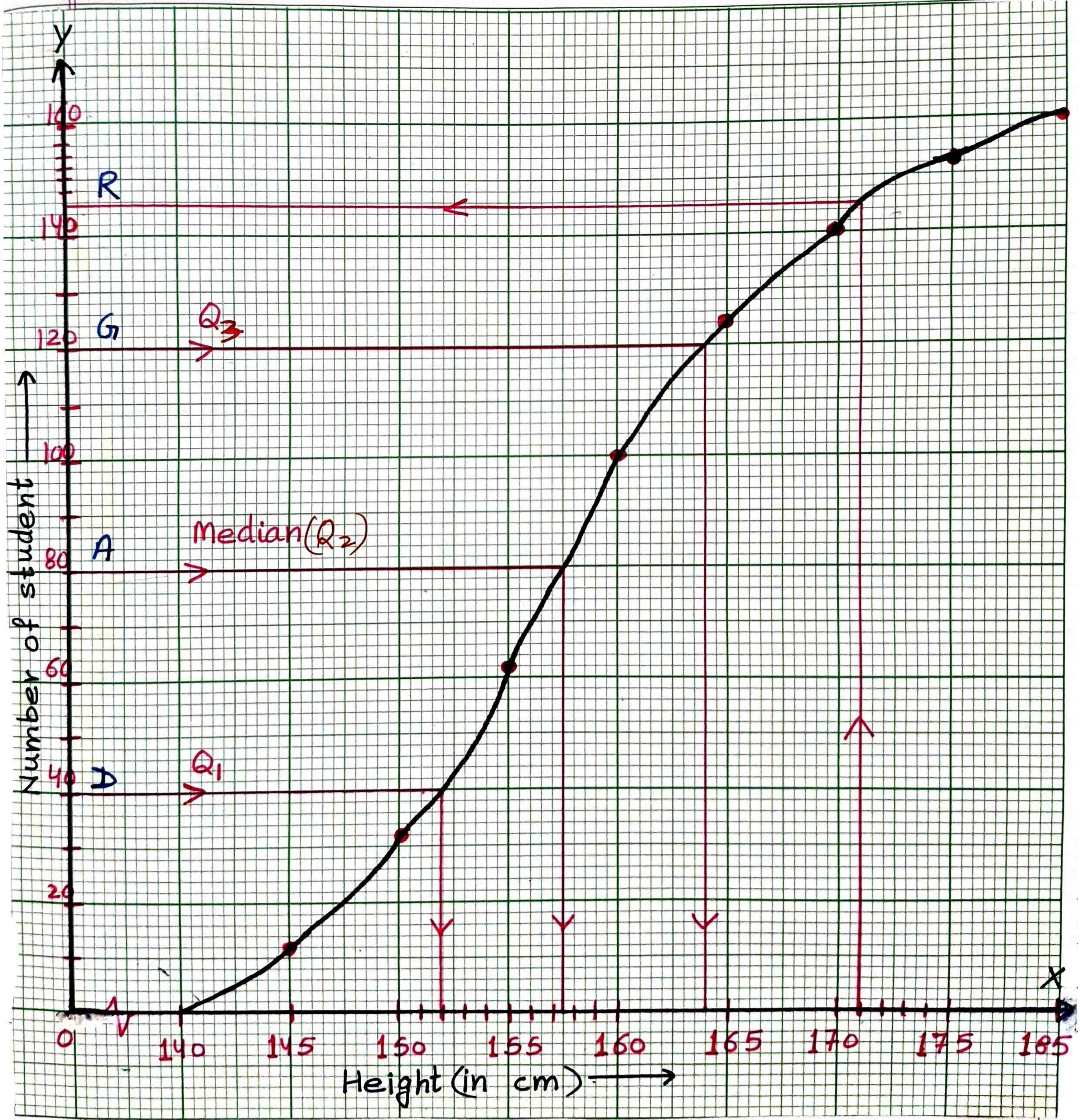
(ii) The Interquartile range

(iii) The number of students whose height is above 172 cm.

Solution :- First prepare the cumulative frequency table.

Marks (in cm)	Number of students	Cumulative frequency
140 - 145	12	12
145 - 150	20	32
150 - 155	30	62
155 - 160	38	100
160 - 165	24	124
165 - 170	16	140
170 - 175	12	152
175 - 180	8	160

Now, we take height along x-axis and the number of students along y-axis.
Taking Scale 2cm = 5 cm height on x-axis and 2cm = 20 students on y-axis.



(i) Here, $N = 160 \Rightarrow \frac{N}{2} = \frac{160}{2} = 80$

On the graph paper, take a point A on the y-axis, representing 80.
Through A, draw a horizontal line, meeting the ogive at B.
From B, draw $BC \perp$ x-axis, meeting it at C.
The abscissa of C is 157.5
Hence, median = 157.5 cm

(ii) Now, $\frac{N}{4} = \frac{160}{4} = 40$

Take a point D on the y-axis, representing 40.
Through D, draw horizontal line, meeting the ogive at E.
From E, draw $EF \perp$ x-axis, meeting it at F.
The abscissa of F is 152. So, $Q_1 = 152$ cm

Also, $\frac{3N}{4} = \frac{3 \times 160}{4} = 120$

So, take a point G on the y-axis, represen 120.
Through G, draw horizontal line, meeting the ogive at H.
From H, draw $HK \perp$ x-axis, meeting it at K.
The abscissa of K is 164. So, $Q_3 = 164$ cm
Therefore, Interquartile range is
$Q_3 - Q_1 = 164 - 152 = 12$ cm

(iii) From ogive, number of students whose height is less than 172 is 145
Number of students whose height is more than 172 = 160 − 145 = 15