Ch 25 Measures of Central Tendency

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Central tendency → Often, we observe that every statistical data has a tendency to cluster around a certain value, possibly at the centre of observations. This tendency of the statistical data is known as its central tendency.

Measures of Central Tendency →

A numerical value which represents the entire statistical data is called a measure of central tendency of that data.

The numerical value which represents the entire statistical data is neither the lowest nor the highest value in the data, rather it lies in between the two extreme values of the data.

The different ways of measuring central tendency of a statistical data are :-

(i) Mean (ii) Median (iii) Mode.

I MEAN (Arithmetic Mean)

The average of numbers in arithmetic is known as the Arithmetic Mean of these numbers in statistics.

Mean of an Ungrouped Data

The Mean of 'n' observations $x_1, x_2, x_3, ---, x_n$ is given by the formula :-

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + --- + x_n}{n} = \frac{\Sigma x_i}{n}$$

where the Σ, called 'sigma' stands for the summation of the terms.

Example 1:- If the mean of 7, 10, 4, 12, x, 3 is 7.5, find the value of x

Solution :- Sum of the given numbers

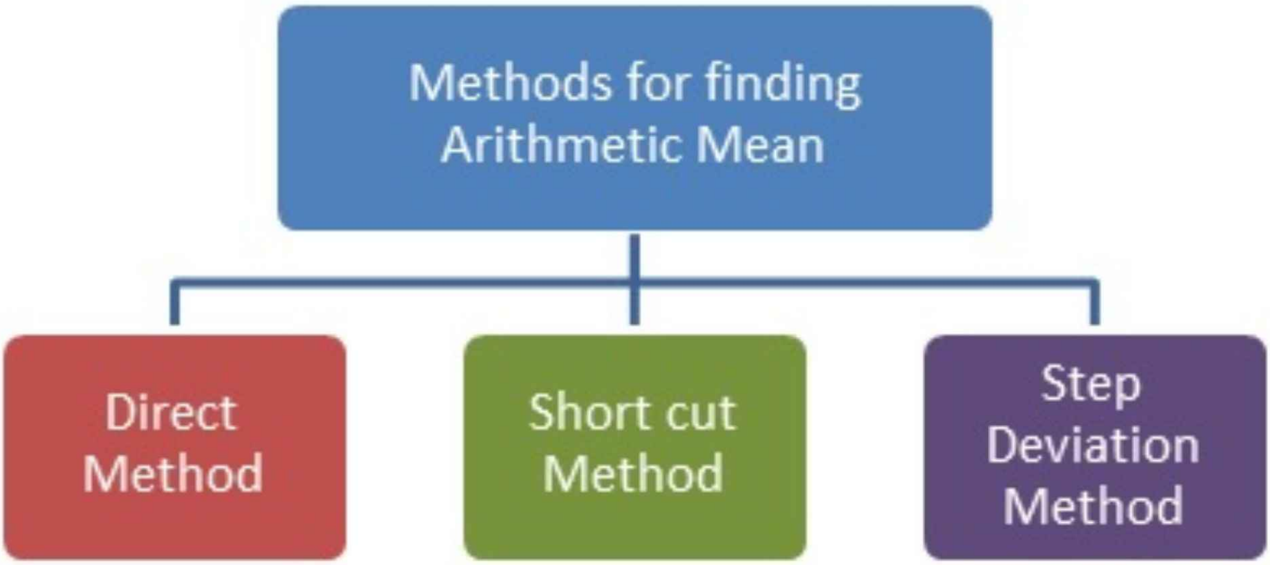
$= 7+10+4+12+x+3 \quad = 36+x$

Number of these terms $= 6$

Given, mean $= 7.5 \quad \Rightarrow \dfrac{36+x}{6} = 7.5$

$\Rightarrow 36+x = 45 \quad \Rightarrow x = 45-36 = 9$

Hence, $x = 9$



Now, let us discuss about mean of Grouped Data

1) **Direct Method :-** When the variates (terms) $x_1, x_2, x_3, ----, x_n$ have frequencies $f_1, f_2, f_3, ----, f_n$ respectively, then the mean is given by the formula :-

$$\text{Mean} = \frac{(f_1x_1 + f_2x_2 + f_3x_3 + --- + f_nx_n)}{(f_1 + f_2 + --- f_n)}$$

$$= \frac{\Sigma f_i x_i}{\Sigma f_i}$$

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Example 2:- The following table shows the weights of 15 members of an athletic team in a school.

Weight (in kg)	42	45	46	48	49
Number of athletes	4	3	5	2	1

Find the mean weight.

Solution :- From the above data, we may prepare the table given below:-

Weight (x_i) (in kg)	Frequency (f_i)	$f_i x_i$
42	4	168
45	3	135
46	5	230
48	2	96
49	1	49
	$\Sigma f_i = 15$	$\Sigma f_i x_i = 678$

$$\text{Mean Weight} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{678}{15} = 45.2 \text{ kg}$$

Hence, the mean weight of the given team is 45.2 kg.

2) Short Cut Method :- Under this method, larger quantities get converted into smaller ones, making the process of multiplication and division easier.

Method :- From the given data, we suitably choose a term, usually the middle term and call it the assumed mean, to be denoted by A.

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So, Mean $= A + \frac{\Sigma f_i d_i}{\Sigma f_i}$

where, deviations, $d_i = x_i - A$ for each term

Example 3 Using short cut method, find the mean from the following data

Variate (x_i)	18	19	20	21	22	23	24
Frequency (f_i)	184	212	327	376	614	372	415

Solution:- Let the assumed mean be, $A = 21$

From the given data, we prepare the table given below:-

Variate (x_i)	Frequency (f_i)	$d_i = x_i - A$ $= x_i - 21$	$f_i d_i$
18	184	−3	−552
19	212	−2	−424
20	327	−1	−327
21	376	0	0
22	614	1	614
23	372	2	744
24	415	3	1245
	$\Sigma f_i = 2500$		$\Sigma f_i d_i = 1300$

Therefore, Mean $= A + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$= 21 + \frac{1300}{2500} = 21 + 0.52$

$= 21.52$

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3) Step Deviation Method → In the previous method of the Assumed Mean Deviation, we calculated $d_i = x_i - A$, where A stands for the assumed mean. In most of the problems, width of all classes is the same, so we can further simplify the calculations of the mean by computing the mean of $u_1, u_2, u_3 \cdots\cdots, u_n$

where, $u_i = \frac{x_i - A}{h}$, h is the with of class.

Then the mean is given by the formula :-

$$\text{Mean} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

Example 4 :- Find the mean of the following distribution by step deviation method:-

Class-intervals	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	10	6	8	12	5	9

Solution:- Let $A = 45$ and $h = 10$

Classes	Classmark (x_i)	$u_i = \frac{x_i - A}{h}$	f_i	$f_i u_i$
20-30	25 = $\frac{20+30}{2}$	-2	10	-20
30-40	35	-1	6	-6
40-50	45	0	8	0
50-60	55	1	12	12
60-70	65	2	5	10
70-80	75	3	9	27
Total			50	23

Now, $\text{Mean} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$$= 45 + \frac{23}{50} \times 10 = 49.6$$

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Example 5:- The marks obtained by 40 students in a short assessment are given below, where 'a' and 'b' are two missing data:-

Marks :	5	6	7	8	9
No. of students :	6	a	16	13	b

If the mean of the distribution is 7.2, find 'a' and 'b'.

Solution:- We have

Marks (x_i)	Number of students (f_i)	$f_i \times x_i$
5	6	30
6	a	6a
7	16	112
8	13	104
9	b	9b
	$\Sigma f_i = 35 + a + b$	$\Sigma f_i x_i = 246 + 6a + 9b$

Given, $\Sigma f_i = 40$

$\Rightarrow 35 + a + b = 40 \Rightarrow a + b = 5$ ------ (i)

Also, Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{246 + 6a + 9b}{40}$

$\Rightarrow \dfrac{246 + 6a + 9b}{40} = 7.2$ [Given, Mean $= 7.2$]

$\Rightarrow 246 + 6a + 9b = 7.2 \times 40$

$\Rightarrow 6a + 9b = 42 \Rightarrow 2a + 3b = 14$ ------ (ii)

Now, solving equation (i) and (ii), we get

$$\begin{array}{r} 2a + 2b = 10 \\ -\ 2a + 3b = 14 \\ \hline -b = -4 \end{array}$$

$\Rightarrow b = 4$

$a = 5 - 4 = 1$

So, $a = 1$ and $b = 4$

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Example 6:- The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Find the values of p and q.

Classes	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	p	10	q	7	8

Solution:- We can use step-deviation method. Construct the table as under, taking assumed mean, $A = 70$. Here, 'h' (width of class) $= 20$

Classes	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
0-20	10	-3	5	-15
20-40	30	-2	p	$-2p$
40-60	50	-1	10	-10
60-80	70	0	q	0
80-100	90	1	7	7
100-120	110	2	8	16
Total			$30+p+q$	$-2p-2$

Since sum of frequencies is 50

$\Rightarrow 30+p+q = 50 \quad \Rightarrow p+q = 20$ ---- (i)

$$\text{Mean} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$\Rightarrow 62.8 = 70 + \frac{(-2p-2)}{50} \times 20$$

$$\Rightarrow \frac{2}{5}(2p+2) = 70 - 62.8$$

$\Rightarrow 4(p+1) = 5 \times 7.2 \quad \Rightarrow p = 8$

Using (i), we get $8+q = 20 \Rightarrow q = 12$

Hence, $p = 8$, $q = 12$