Class 10, Mathematics

Date 8.7.2024

Chapter 13 Section and Mid-point Formulae

Ms. Reena

SECTION FORMULA

For any two known (given) points in a co-ordinate (Cartesian) plane, the knowledge of co-ordinate geometry may be used to find :-

(i) the distance between the given points

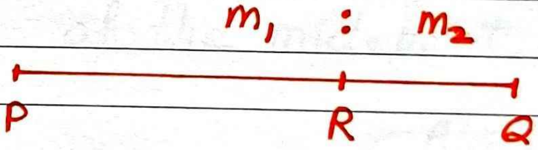
(ii) the co-ordinates of a point which divides the line joining the given points in a given ratio,

(iii) the co-ordinates of the mid-point of the line segment joining the two given points

(iv) equation of the straight line through the given points.

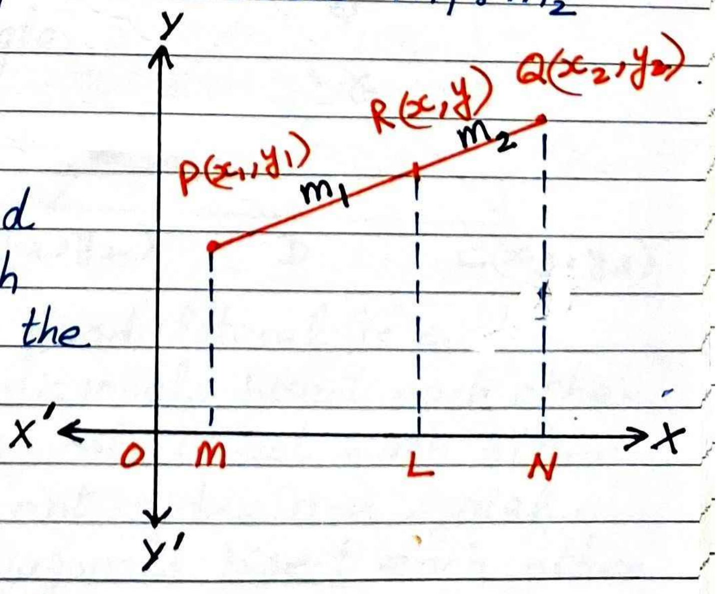
(v) equation of the perpendicular bisector of the line segment obtained on joining the given two points, etc.

Let P, Q be two points in a plane and R be a point on the line segment joining points P and Q such that $PR : RQ = m_1 : m_2$ then we say that the point R divides the line segment PQ internally in the ratio $m_1 : m_2$



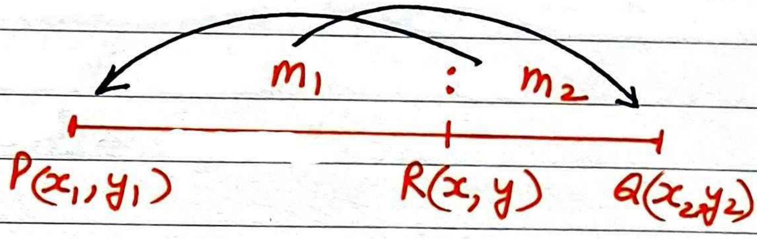
Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two given points in the co-ordinate plane, and $R(x, y)$ be the point which divides PQ (internally) in the given ratio $m_1 : m_2$

i.e. $\dfrac{PR}{RQ} = \dfrac{m_1}{m_2}$



Ms. Reena

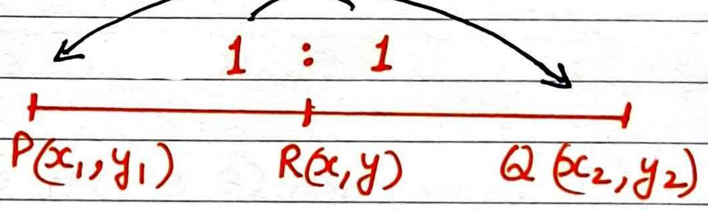
Section Formula



$$x = \frac{m_1x_2 + m_2x_1}{m_1+m_2}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1+m_2}$$, where m_1+m_2 is sum of ratio

Mid point Formula



PR = RQ i.e. 1 : 1

$\Rightarrow m_1 : m_2 = 1 : 1$

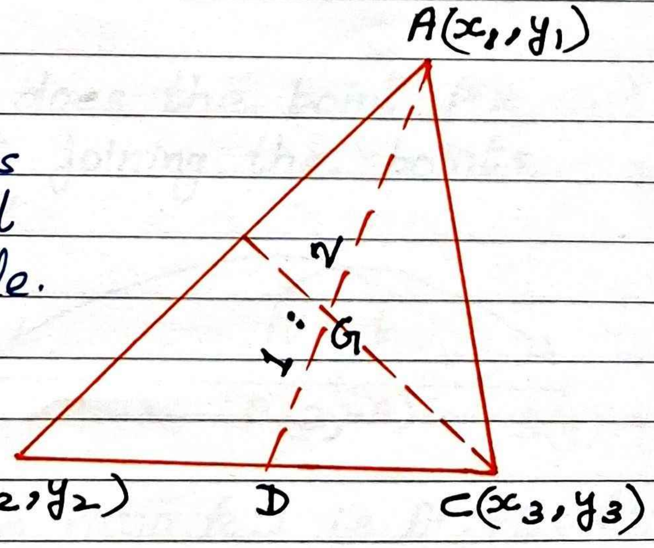
$$x = \frac{1\times x_1 + 1\times x_2}{1+1} = \frac{x_1+x_2}{2}$$

$$y = \frac{1\times y_1 + 1\times y_2}{1+1} = \frac{y_1+y_2}{2}$$

Hence, the co-ordinates of the mid-point of PQ are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Centroid of a triangle

The point where the medians of a triangle meet is called the centroid of the triangle.



The coordinates of G are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Note :– To prove that a quadrilateral is a

(i) Parallelogram : show that diagonals bisect each other

(ii) Rhombus : show that diagonals bisect each other and two adjacent sides are equal.

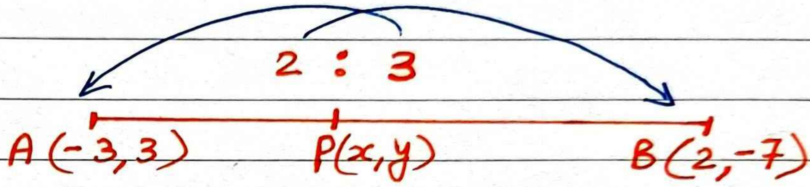
(iii) Rectangle : show that diagonals bisect each other and are equal.

(iv) Square : show that diagonals bisect each other and two adjacent sides are equal and diagonals are also equal.

Example 1 Calculate the coordinates of the point P which divides the line segment joining A (−3, 3) and B (2, −7) internally in the ratio 2:3.

Solution:-

Let (x, y) be the coordinates of the point P which divides



the line segment joining A (−3, 3) and B(2, −7) internally in the ratio 2:3, then

$$x = \frac{2 \times 2 + 3 \times (-3)}{2+3} = \frac{4-9}{5} = \frac{-5}{5} = -1$$

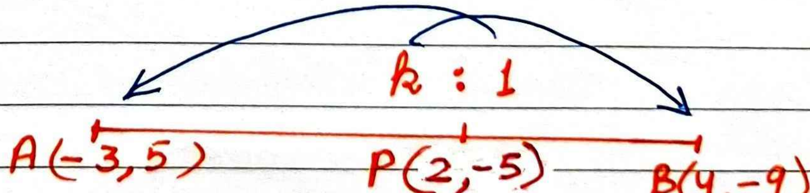
$$y = \frac{2 \times (-7) + 3 \times 3}{2+3} = \frac{-14+9}{5} = \frac{-5}{5} = -1$$

Therefore coordinates of P are (−1, −1)

Example 2 In what ratio does the point P(2, −5) divide the line segment joining the points A (−3, 5) and B(4, −9) ?

Solution:-

Let P (2, −5) divide the line segment joining the points



A(−3, 5) and B(4, −9) in the ratio $k:1$ i.e. $AP : PB = k:1$

Therefore, coordinates of P are

$$\left(\frac{k \times 4 + 1 \times (-3)}{k+1}, \frac{k \times (-9) + 1 \times 5}{k+1} \right) \quad \text{But P is } (2, -5)$$

$$\Rightarrow \frac{4k-3}{k+1} = 2 \quad \text{----(i)} \quad \text{and} \quad \frac{-9k+5}{k+1} = -5 \quad \text{----(ii)}$$

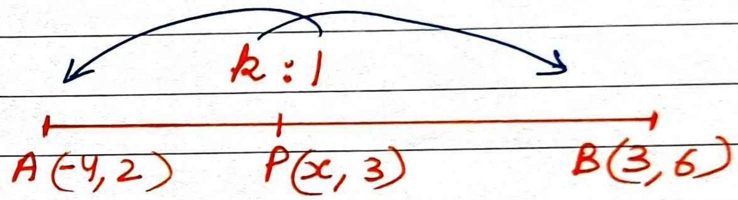
Solving equation (i), we get

$\frac{4k-3}{k+1} = 2 \Rightarrow 4k-3 = 2k+2 \Rightarrow 2k = 5 \Rightarrow k = \frac{5}{2}$

So, required ratio is $\frac{5}{2}:1$ i.e. $5:2$

Remark:- On solving equation (ii), we get same result. So, we can solve any one equation to get value of 'k'

Example 3 Calculate the ratio in which the line joining $A(-4,2)$ and $B(3,6)$ is divided by point $P(x,3)$. Also find x.



Solution:- Let $P(x,3)$ divide the line segment joining $A(-4,2)$ and $B(3,6)$ in the ratio $k:1$ i.e. $AP:PB = k:1$

The point P is $\left(\frac{3k-4}{k+1}, \frac{6k+2}{k+1}\right)$ But P is $(x,3)$

$\Rightarrow \frac{3k-4}{k+1} = x$ and $\frac{6k+2}{k+1} = 3$

$6k+2 = 3k+3$

$3k = 1$

$k = \frac{1}{3}$

$\Rightarrow 3k-4 = x(k+1)$

Putting $k = \frac{1}{3}$

$3 \times \frac{1}{3} - 4 = x\left(\frac{1}{3}+1\right)$

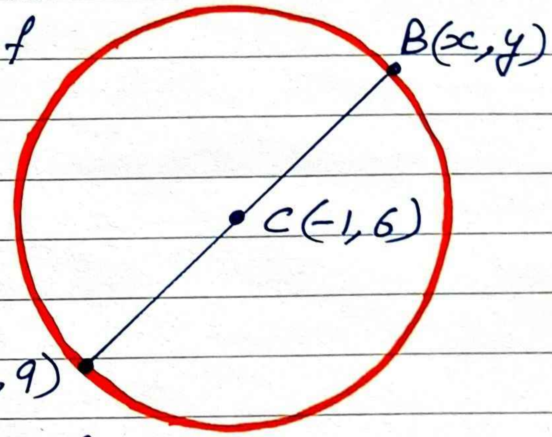
$-3 \times \frac{3}{4} = x \Rightarrow x = -\frac{9}{4}$

Hence, $k = \frac{1}{3}$ and $x = -\frac{9}{4}$

Therefore, ratio $= \frac{1}{3}:1$ i.e. $1:3$
and coordinate of P is $\left(-\frac{9}{4}, 3\right)$

Example 4 The centre of a circle is $C(-1,6)$ and one end of a diameter is $A(5,9)$. Find the co-ordinates of the other end.

Solution:- Let the other end of the diameter of the circle be $B(x,y)$ whose one end is the point $A(5,9)$



The mid-point of AB is

$\frac{5+x}{2}, \frac{9+y}{2}$

The centre of the circle is $C(-1,6)$

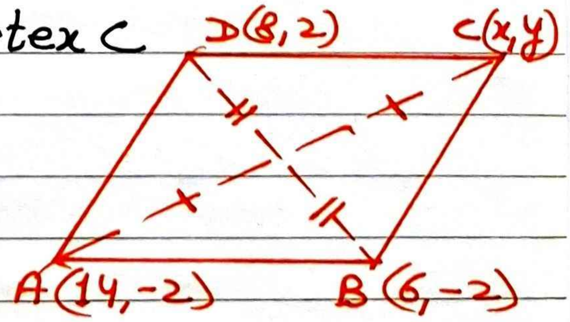
Since the centre of the circle is the mid-point of AB, $\Rightarrow \frac{5+x}{2} = -1$ and $\frac{9+y}{2} = 6$

$\Rightarrow 5+x = -2$ and $9+y = 12$

$\Rightarrow x = -7$ and $y = 3$

Therefore, coordinates of the other end of the diameter are $(-7,3)$

Example 10:- $A(14,-2)$, $B(6,-2)$ and $D(8,2)$ are the three vertices of a parallelogram ABCD. Find the co-ordinates of the fourth vertex C



Solution:- Let $C = (x,y)$

Since the diagonals of a parallelogram bisect each other.

$\therefore$ Mid-point of AC = mid point of BD

$$\Rightarrow \left(\frac{14+x}{2}, \frac{-2+y}{2}\right) = \left(\frac{8+6}{2}, \frac{2+(-2)}{2}\right)$$

$$\Rightarrow \frac{14+x}{2} = \frac{14}{2} \text{ and } \frac{-2+y}{2} = \frac{0}{2} \Rightarrow x=0 \text{ and } y=2$$

Hence, vertex $C = (0,2)$