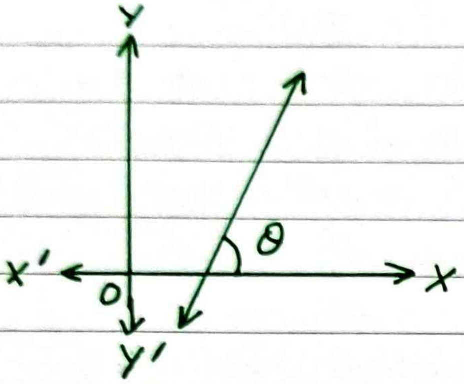
Tender Heart High School,Sector 33B,Chd.

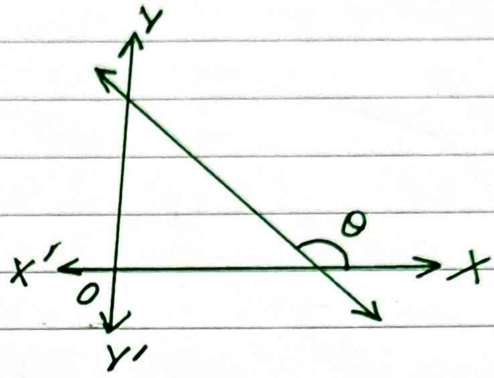
Class 10th Mathematics Date 15.7.2024

Ch - 14 Equation of a Straight Line

Ms. Reena

INCLINATION OF A STRAIGHT LINE





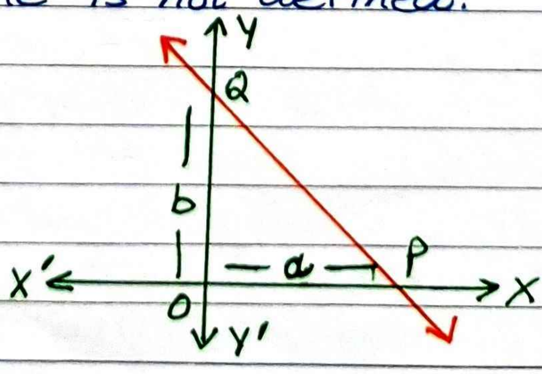
The angle which a straight line makes with the positive direction of x-axis measured in the anticlockwise direction is called the inclination (or angle of inclination) of the line.
The inclination is usually denoted by θ.

→ Inclination of a line parallel to y-axis is 90°
→ Inclination of a line parallel to x-axis is 0°
(i) Any line parallel to x-axis is horizontal line.
(ii) Any line parallel to y-axis is vertical line.
(iii) A line which is neither parallel to x-axis nor parallel to y-axis is called an oblique line.

Slope (or gradient) of a straight line

→ The slope of a line is usually denoted by 'm'
If $\theta\,(\neq 90^\circ)$ is the inclination of a line then
$m = \tan\theta$
If $\theta = 90^\circ$, then $\tan 90^\circ$ is not defined, i.e. slope of a verticle line is not defined.

Here, a is x-intercept
b is y-intercept
Co-ordinates of P (a, 0) and Q (0, b)



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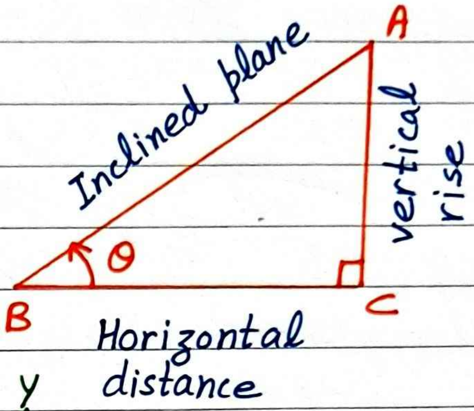
slope of AC

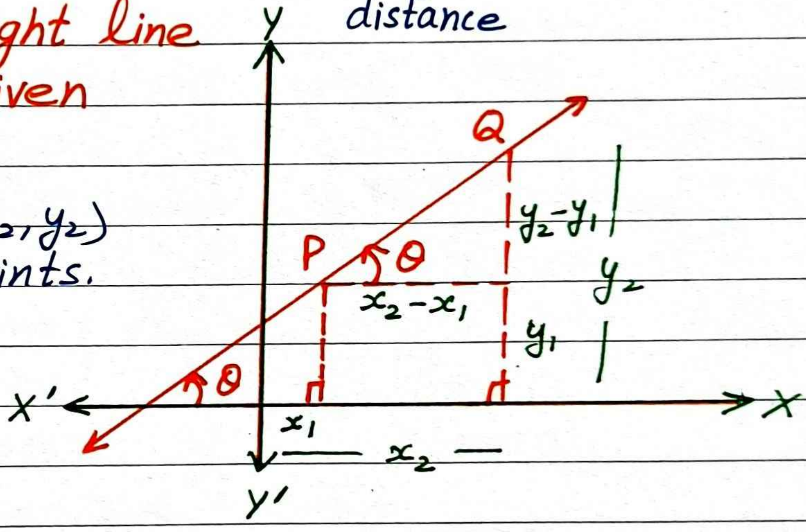
$= \dfrac{\text{vertical rise}}{\text{horizontal distance}} = \dfrac{AB}{BC}$

$= \tan\theta$



The slope of a straight line passing through two given fixed points :-

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two fixed points.

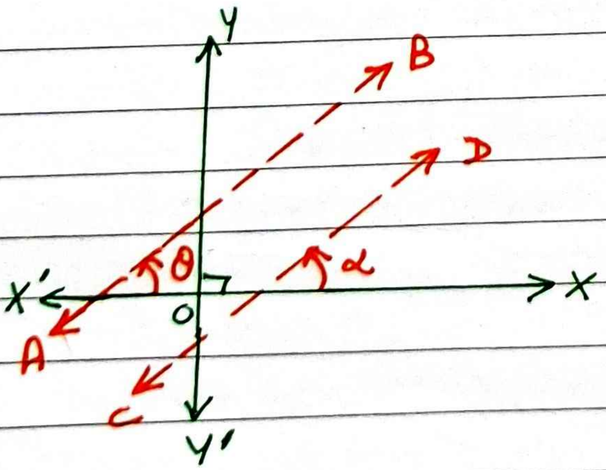


slope 'm' $= \tan\theta$

$= \dfrac{y_2 - y_1}{x_2 - x_1}$ or $\dfrac{y_1 - y_2}{x_1 - x_2}$

i.e. $\dfrac{\text{Difference of ordinates of the given points}}{\text{Difference of their abscissae}}$

Parallel Lines

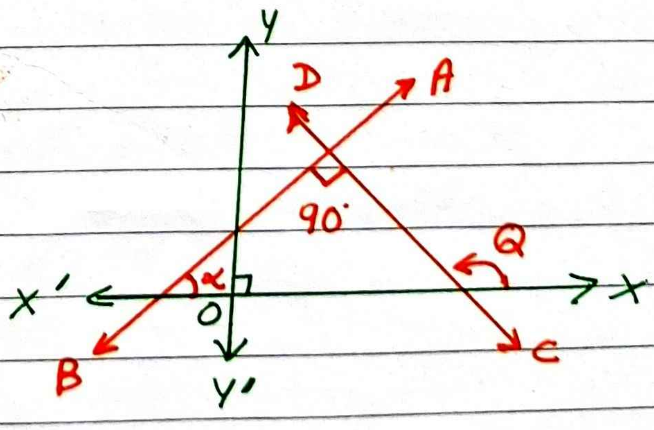


$\tan\theta = \tan\alpha$

slope of AB = slope of CD

i.e. $m_1 = m_2$

Perpendicular Lines



$\tan\theta \cdot \tan\alpha = -1$

slope of AB × slope of CD = -1

i.e. $m_1 \cdot m_2 = -1$

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Hence, if two lines are parallel their slopes are equal.
If two lines are mutually perpendicular, the product of their slopes is -1.

Example 1 Find the slope of the line passing through the points $A(-2,3)$ and $B(2,7)$. Also find:-
(i) the inclination of the line AB
(ii) slope of the line parallel to AB
(iii) slope of the line perpendicular to AB.

Solution:- Let $A(-2,3) = (x_1, y_1)$ and $B(2,7) = (x_2, y_2)$

Slope of AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-3}{2+2} = 1$

(i) Let inclination of line AB be θ

$\therefore \tan\theta = 1 \Rightarrow \tan\theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$

(ii) Slope of the line parallel to AB
$=$ slope of AB $= 1$

(iii) Slope of the line perpendicular to AB

$= -\dfrac{1}{\text{slope of AB}} = -\dfrac{1}{1} = -1$

Example 2 The line joining $A(-3,4)$ and $B(2,-1)$ is parallel to the line joining $C(1,-2)$ and $D(0,x)$. Find x

Solution:- slope of AB = slope of CD [Since $AB \parallel CD$]

$\Rightarrow \dfrac{-1-4}{2+3} = \dfrac{x+2}{0-1}$

$\Rightarrow x = -1$

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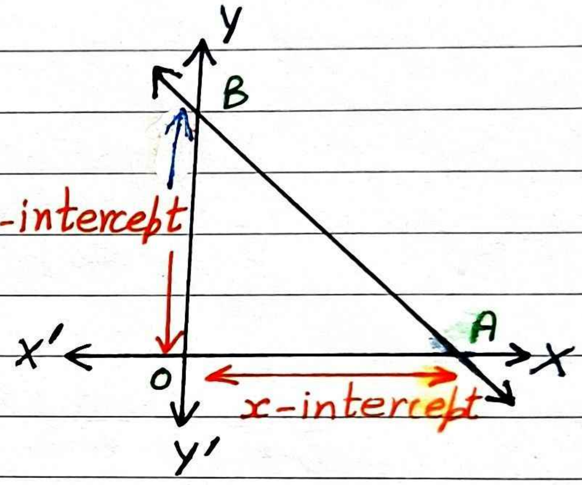
Equation of a Line :-

The equation of a straight line is a linear equation in x and y, which is satisfied by the co-ordinates of every point on the line and not by any point outside this line.

X-intercept

If a line meets the x-axis at a point A, then the distance of point A from the origin O (i.e. OA) is called x- intercept.



Y- intercept

If a line meets the y-axis at point B, then the distance of point B from the origin O (i.e. OB) is called y- intercept.

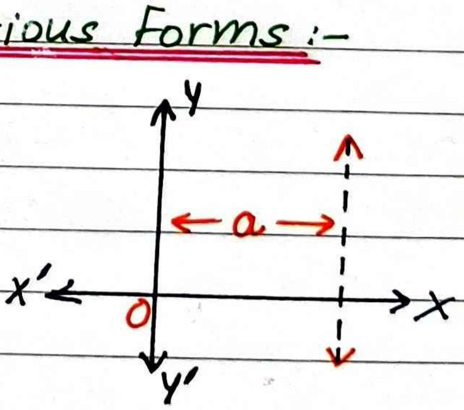
e.g. If x-intercept of a point A is 5 then A(5,0)
If y- intercept of a point B is 4 then B(0,5)
If y-intercept of a point P is -8 then P(0,-8)

Equation of a Straight Line in Various Forms :-

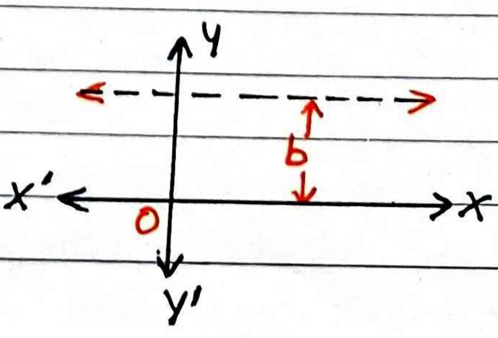
I Equation of x-axis is $y=0$

II Equation of y-axis is $x=0$

III Equation of a line parallel to y-axis, and at a distance 'a' from it is $x=a$



IV Equation of a line parallel to x-axis, at a distance 'b' from it is $y=b$



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Equation of an Oblique Line in Various Forms

I Slope-intercept Form

When slope (gradient) and intercept on the y-axis are given.

Let AB be a line which makes an angle $\theta°$ with x-axis and whose intercept OB on the y-axis is 'c' (i.e. $OB = c$)

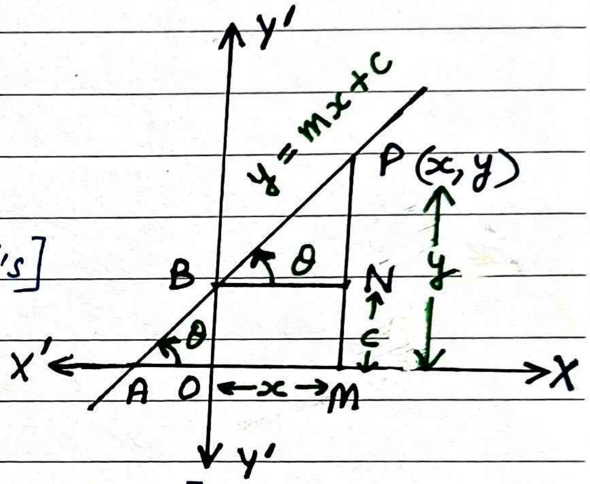
Then, $m = \tan\theta$

Let $P(x, y)$ be any point on AB

Draw $PM \perp$ x-axis and $BN \perp PM$

Then, clearly $BN \parallel$ x-axis

$\therefore \angle PBN = \angle BAO = \theta$ [corresponding $\angle$'s]



Now, $BN = OM = x$, $MN = OB = c$

and $PM = y$

From right-angled ΔBNP, we have

$$\frac{PN}{BN} = \tan\theta \Rightarrow \frac{PM - MN}{BN} = m \quad [\because \tan\theta = m]$$

$$\Rightarrow \frac{y - c}{x} = m \quad [\because PM = y, MN = OB = c \text{ and } BN = OM = x]$$

$$\Rightarrow y = mx + c$$

Hence, the equation of the given line is $y = mx + c$.

II Point-slope Form

The equation of a line with slope m and passing through a point $A(x_1, y_1)$ is given by,

$$(y - y_1) = m(x - x_1)$$

III Two-point Form

The equation of a line passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

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Example 1 Find the equation of a line :-

(i) whose inclination is 45° and y-intercept is 5

Solution:- Given, inclination is 45° and y-intercept $c = 5$,

$\therefore$ slope $m = \tan\theta = \tan 45° = 1$

Substituting the values of m and c in the equation $y = mx + c$, we get $y = x + 5$

(ii) with inclination = 60° and passing through (-2, 5)

Given, $\theta = 60°$

$\Rightarrow$ slope $m = \tan 60° = \sqrt{3}$ and $(x_1, y_1) = (-2, 5)$

Substituting in $y - y_1 = m(x - x_1)$, we get

$y - 5 = \sqrt{3}(x + 2) \Rightarrow y = \sqrt{3}x + 2\sqrt{3} + 5$

(iii) passing through the points (-3, 1) and (1, 5)

Let $(-3, 1) = (x_1, y_1)$ and $(1, 5) = (x_2, y_2)$

$\therefore$ slope of the line $= m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - 1}{1 + 3} = 1$

Equation is.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 1(x + 3) \Rightarrow y = x + 4$$

Example 2 Find the equation of the line whose x-intercept is 8 and y-intercept is -12.

Solution:- When x-intercept = 8; corresponding point on the x-axis = (8, 0)

When y-intercept = -12; corresponding point on the y-axis = (0, -12)

Let $(8, 0) = (x_1, y_1)$ and $(0, -12) = (x_2, y_2)$

$\Rightarrow$ Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-12 - 0}{0 - 8} = \dfrac{3}{2}$

And, the required equation is $y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = \dfrac{3}{2}(x - 8) \Rightarrow 2y = 3x - 24$

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Example 3 Find the equation of the line which passes through (2, 7) and whose y-intercept is 3.

Solution :- y-intercept = 3

$\Rightarrow$ The corresponding point on the y-axis = (0, 3)

Now, we have $(x_1, y_1) = (2, 7)$ and $(x_2, y_2) = (0, 3)$

$\therefore$ slope (m) $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-7}{0-2} = \dfrac{-4}{-2} = 2$

And, required equation is $y - y_1 = m(x - x_1)$

$\Rightarrow y - 7 = 2(x-2) \Rightarrow y - 7 = 2x - 4$

$\Rightarrow y = 2x + 3$

Alternative method :-

Given y-intercept = 3 i.e. c = 3

$\therefore y = mx + c \Rightarrow y = mx + 3$

Since, the line $y = mx + 3$ passes through (2, 7)

$\therefore 7 = m \times 2 + 3 \Rightarrow m = 2$

Hence, the required equation is $y = 2x + 3$

Find the equation of a line given a point on the line and the slope (gradient)

Example:

Find the equation of the line passing through (-3,1) with slope 2.

Using

$y = mx + c$

$y = 2x + c$ m = slope = 2

1 = 2(-3) + c

1 = -6 + c

c = 7

Substitute x = -3 and y = 1

Equation is:

$y = 2x + 7$

$2x - y + 7 = 0$

Using

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - -3)$ m = slope = 2

$y - 1 = 2x + 6$

$2x - y + 7 = 0$

Substitute x_1 = -3 and y_1 = 1

1. Distance Formula : The distance between the points P (x_1, y_1) and Q (x_2, y_2)

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. Section Formula : The co-ordinates of the point which divides the line segment joining the points P (x_1, y_1) and Q (x_2, y_2) in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

3. Mid-point Formula : The co-ordinates of the mid-point of the line segement joining the points P (x_1, y_1) and Q (x_2, y_2) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

4. Centroid Formula : The co-ordinates of the centroid of a triangle whose vertices are A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

5. **Slope of a Straight Line :**

(i) If the inclination of a line is θ ($\neq 90^\circ$), its slope = $m = \tan\theta$.

(ii) Slope of a line through (x_1, y_1) and (x_2, y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

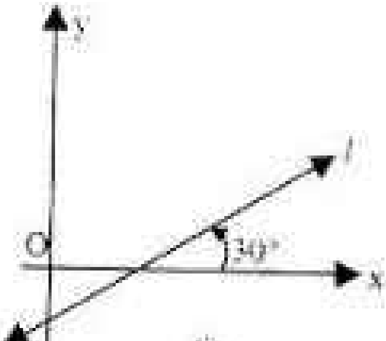
6. **Equation of a Straight Line :**

(i) Equation of a line parallel to x-axis is $y = b$.

(ii) Equation of a line parallel to y-axis is $x = a$.

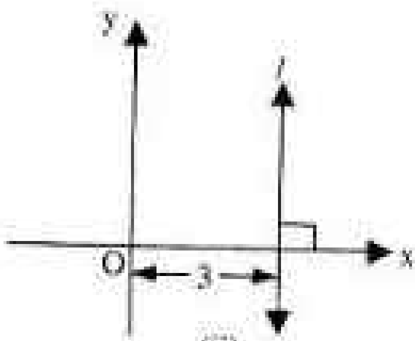
(iii) Equation of a line with slope m and y-intercept c is $y = mx + c$.

(iv) Equation of a line through (x_1, y_1) and with slope m is $y - y_1 = m(x - x_1)$.



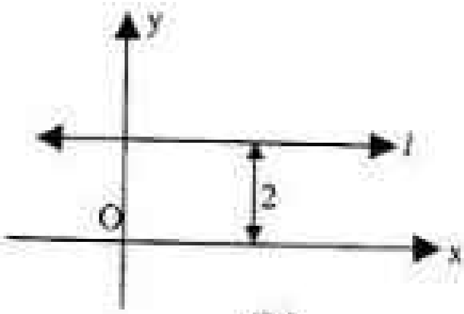
(i)

Slope of line $l = \tan 30^\circ = \frac{1}{\sqrt{3}}$



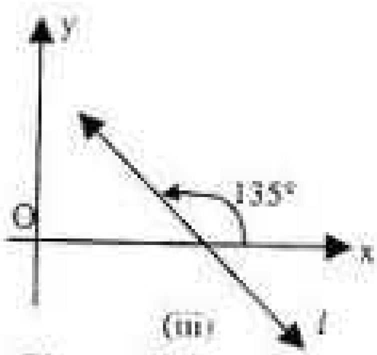
(ii)

Slope of line $l = \tan 90^\circ\ \infty$



(iv)

Slope of line $l = \tan 0 = 0$



(iii)

Slope of line $l = \tan 135^\circ = -1$