Class : 10th Subject : Mathematics Date: 8.04.2024

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Topic : Chapter-5 Quadratic Equations

Introduction : The word "quadratic" comes from Latin word "quadratum" means square. Hence, we define a quadratic equation as an equation where the variable is of the second degree. Therefore, a quadratic equation is also called an "Equation of degree 2".

We know that a quadratic polynomial can be written as ax^2+bx+c

If $ax^2+bx+c=0$, it form a quadratic equation.

Variable : Letters of English alphabet e.g. $a, b, c, \ldots x, y, z$

Constant : A number with a definite value e.g. $1, 5, 12, 82, \ldots$

Equation : A mathematical statement that two expressions are equal. e.g. $4+6=10$, $2x+5=15$, $3x^2-x+5=0$ etc.

Standard form of a quadratic equation

$$ax^2+bx+c=0$$

where, x is an unknown variable, and a, b, c are constants with $(a \neq 0)$. If $(a=0)$, then it becomes a linear equation. The constant 'a', 'b' and 'c' are called the coefficients. Note: [$a \neq 0$ but 'b' and 'c' can be equal to zero]

Examples

1) $2x^2+5x+3=0$ Here, $a=2$, $b=3$, $c=5$

2) $x^2-3x=0$ Here, $a=1$, $b=-3$, $c=0$

3) $x^2-36=0$ Here, $a=1$, $b=0$, $c=-36$

Quadratic equation can be solved by following methods:-

1) Factorisation 2) completing the square

3) Graphing 4) Newton's method 5) quadratic formula

In this chapter we shall learn only two methods i.e. Factorisation and quadratic formula as other methods are not in ICSE class X maths syllabus.

I Solving a quadratic equation by Factorisation

STEPS:-

1) Clear all fractions and brackets (if any)
2) Write the given equation in the form $ax^2+bx+c=0$
3) Factorise the left side into product of two linear factors.
4) Put each factor equal to zero and solve.

[Zero Product Rule: If a and b be any two real numbers Then, $ab=0 \Rightarrow a=0$ or $b=0$]

Example 1 Solve the following equations

(i) $2x^2 = 3x \Rightarrow 2x^2 - 3x = 0 \Rightarrow x(2x-3) = 0$

$\Rightarrow x = 0$ or $2x-3=0$ [Zero Product Rule]

$\Rightarrow x = 0$ or $x = \frac{3}{2}$

Hence, the roots of the given equation are $\{0, \frac{3}{2}\}$

(ii) $(x+3)(x-3) = 40$

$\Rightarrow x^2 - 9 = 40 \Rightarrow x^2-9-40=0 \Rightarrow x^2-49=0$

$\Rightarrow x^2 - 7^2 \Rightarrow (x+7)(x-7) = 0$

$\Rightarrow x+7=0$ or $x-7=0 \Rightarrow x=-7$ or $x=7$

Hence, the roots of the given equation are $\{-7, 7\}$

(iii) $\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}$

$\Rightarrow \frac{x^2+(x-1)^2}{x(x-1)} = \frac{5}{2} \Rightarrow 2(x^2+x^2-2x+1) = 5(x^2-x)$

$\Rightarrow 4x^2-4x+2 = 5x^2-5x \Rightarrow -x^2+x+2 = 0$

$\Rightarrow x^2-x-2 = 0 \Rightarrow x^2-2x+x-2 = 0$

$\Rightarrow x(x-2)+1(x-2) = 0 \Rightarrow (x-2)(x+1)$

$\Rightarrow x-2=0$ or $x+1=0 \Rightarrow x=2$ or $x=-1$

Hence, the roots of the given equation are $2, -1$

(iv) $3(2x-1)^2 + 4(2x-1) - 4 = 0$

Put $2x-1 = y$, we get $3y^2+4y-4=0$

$\Rightarrow 3y^2+6y-2y-4=0 \Rightarrow 3y(y+2)-2(y+2)=0$

$\Rightarrow (y+2)(3y-2)=0 \Rightarrow y+2=0$ or $3y-2=0$

$\Rightarrow y=-2$ or $y=\frac{2}{3}$

When $y = -2$, $2x - 1 = -2 \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$

When $y = \frac{2}{3}$, $2x - 1 = \frac{2}{3} \Rightarrow 2x = \frac{2}{3} + 1 = \frac{5}{3} \Rightarrow x = \frac{5}{6}$

Hence, the roots are $\left\{-\frac{1}{2}, \frac{5}{6}\right\}$

II Solving quadratic equations using the formula:-

We know that quadratic equation is $ax^2 + bx + c = 0$

Multiplying both sides by $4a$, we get

$4a^2x^2 + 4abx + 4ac = 0$

$\Rightarrow (2ax)^2 + 2 \cdot 2ax \cdot b = -4ac$ Adding b^2 on both sides,

we get, $(2ax)^2 + 2 \cdot 2ax \cdot b + b^2 = b^2 - 4ac$

$\Rightarrow (2ax + b)^2 = b^2 - 4ac \Rightarrow 2ax + b = \pm\sqrt{b^2 - 4ac}$

$\Rightarrow 2ax = -b \pm \sqrt{b^2 - 4ac} \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So, Quadratic Formula (Shreedharacharya's Rule)

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where symbol '$\pm$' indicates two solution/roots

Note: The above derivation of formula is not a part of syllabus and there are many different methods to derive the quadratic formula.

Example: Solve the following equations by using quadratic formula:-

(i) $3x^2 - 4x - 4 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we get $a = 3$, $b =$ and $c =$

Applying the formula, we get $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times (-4)}}{2 \times 3}$

$= \dfrac{4 \pm \sqrt{16 + 48}}{6} = \dfrac{4 \pm \sqrt{64}}{6} = \dfrac{4 \pm 8}{6}$

$= \dfrac{4+8}{6}, \dfrac{4-8}{6} = \dfrac{12}{6}, \dfrac{-4}{6} = 2, -\dfrac{2}{3}$

Hence, roots (solution) of the given equation are $2, -\frac{2}{3}$

$\therefore$ Solution Set $= \left\{2, -\frac{2}{3}\right\}$

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Examples

Example 1

(i) $x(2x+5)=0 \Rightarrow x=0$ or $2x+5=0 \Rightarrow x=0$ or $x=\frac{-5}{2}$

Therefore, solution set $=\left\{0, -\frac{5}{2}\right\}$

(ii) $(2x-1)(2x+3)=0 \Rightarrow 2x-1=0$ or $2x+3=0$

$\Rightarrow x=\frac{1}{2}$ or $x=\frac{-3}{2}$ Therefore, solution set $=\left\{\frac{1}{2}, -\frac{3}{2}\right\}$

(iii) $4x^2-1=0 \Rightarrow 4x^2=1 \Rightarrow x^2=\frac{1}{4} \Rightarrow x=\pm\frac{1}{2}$

Therefore, solution set $=\left\{\frac{1}{2}, -\frac{1}{2}\right\}$

Example 2

(i) $x-\frac{4}{x}=0 \Rightarrow \frac{x^2-4}{x}=0 \Rightarrow x^2-4=0 \Rightarrow x^2=4$

$\Rightarrow x=\pm 2$ Therefore, solution set $=\{-2, 2\}$

(ii) $\frac{z}{z-1}-\frac{z}{2}=1 \Rightarrow \frac{2z-z(z-1)}{2(z-1)}=1$

$\Rightarrow 2z-z^2+z=2z-2 \Rightarrow -z^2+z+2=0$

$\Rightarrow z^2-z-2=0 \Rightarrow z^2-2z+z-2=0$

$\Rightarrow z(z-2)+1(z-2)=0 \Rightarrow (z-2)(z+1)=0$

$\Rightarrow z-2=0$ or $z+1=0 \Rightarrow z=2$ or $z=-1$

Therefore, solution set $=\{2, -1\}$

(iii) $\frac{5}{x-2}+\frac{4}{x-1}=7 \Rightarrow \frac{5(x-1)+4(x-2)}{(x-2)(x-1)}=7$

$\Rightarrow 5x-5+4x-8=7(x^2-x-2x+2)$

$\Rightarrow 9x-13=7x^2-21x+14 \Rightarrow -7x^2+30x-27=0$

$\Rightarrow 7x^2-30x+27=0 \Rightarrow 7x^2-21x-9x+27=0$

$\Rightarrow 7x(x-3)-9(x-3)=0 \Rightarrow (x-3)(7x-9)=0$

$\Rightarrow x-3=0$ or $7x-9=0 \Rightarrow x=3$ or $x=\frac{9}{7}$

Therefore, solution set $=\left\{3, \frac{9}{7}\right\}$

(iv) $\frac{x^2+3}{x+1} = 2 \Rightarrow x^2+3 = 2(x+1) \Rightarrow x^2+3 = 2x+2$

$\Rightarrow x^2-2x+1=0 \Rightarrow x^2-x-x+1=0 \Rightarrow x(x-1)-1(x-1)$

$\Rightarrow (x-1)(x-1)=0 \Rightarrow x-1=0$ or $x-1=0$

$\Rightarrow x=1$ or $x=1$ Therefore, solution set $= \{1, 1\}$

example 3

(i) $2\left(\frac{x-1}{2x+1}\right)^2 - 5\left(\frac{x-1}{2x+1}\right) = 12$ Put $\frac{x-1}{2x+1} = y$

$\Rightarrow 2y^2-5y-12=0 \Rightarrow 2y^2-8y+3y-12=0$

$\Rightarrow 2y(y-4)+3(y-4)=0 \Rightarrow (y-4)(2y+3)=0$

$\Rightarrow y-4=0$ or $2y+3=0 \Rightarrow y=4$ or $y=-\frac{3}{2}$

Case I: When $y=4$

$\Rightarrow \frac{x-1}{2x+1} = 4 \Rightarrow x-1 = 4(2x+1) \Rightarrow x-1 = 8x+4 \Rightarrow -7x = 5$

$\Rightarrow x = -\frac{5}{7}$ Case II: When $y = -\frac{3}{2}$

$\Rightarrow \frac{x-1}{2x+1} = -\frac{3}{2} \Rightarrow 2(x-1) = -3(2x+1) \Rightarrow 2x-2 = -6x-3$

$\Rightarrow 2x+6x = -3+2 \Rightarrow 8x = -1 \Rightarrow x = -\frac{1}{8}$

Therefore, solution set $= \left\{-\frac{3}{2}, -\frac{1}{8}\right\}$

example 4

(i) $2(y+1)^2 - 5(y+1) = 12$ Put $y+1 = p$

$\Rightarrow 2p^2-5p-12=0$ Here, $a=2$, $b=-5$, $c=-12$

Applying the formula,

$$p = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2-4\times 2\times(-12)}}{2\times 2}$$

$$= \frac{5 \pm \sqrt{25+96}}{4} = \frac{5 \pm \sqrt{121}}{4} = \frac{5 \pm 11}{4} = \frac{5+11}{4}, \frac{5-11}{4}$$

$$= \frac{16}{4}, -\frac{6}{4} = 4, -\frac{3}{2}$$

Case I: When $p=4 \Rightarrow y+1=4 \Rightarrow y=3$

Case II: When $p=-\frac{3}{2} \Rightarrow y+1 = -\frac{3}{2} \Rightarrow y = -\frac{3}{2}-1 = -\frac{5}{2}$

Therefore solution set $= \left\{3, -\frac{5}{2}\right\}$

(ii) $x - 2\sqrt{x} - 6 = 0$ Put $\sqrt{x} = y \Rightarrow x = y^2$

$\Rightarrow y^2-2y-6=0$ Here, $a=1$, $b=-2$, $c=-6$

Applying the formula,

$$y = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2-4\times 1\times(-6)}}{2\times 1}$$

$= \dfrac{2 \pm \sqrt{4+24}}{2} = \dfrac{2 \pm \sqrt{28}}{2} = \dfrac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$

Case I When $y = 1+\sqrt{7} \Rightarrow \sqrt{x} = 1+\sqrt{7} \Rightarrow x = (1+\sqrt{7})^2$

$\Rightarrow x = 1 + 7 + 2\sqrt{7} = 8 + 2\sqrt{7}$

Case II When $y = 1-\sqrt{7} \Rightarrow \sqrt{x} = 1-\sqrt{7} \Rightarrow x = (1-\sqrt{7})^2$

$\Rightarrow x = 1+7-2\sqrt{7} = 8-2\sqrt{7}$

Solution set $= \{8+2\sqrt{7},\ 8-2\sqrt{7}\}$

NOTE : Here use value of $\sqrt{7} = 2.646$ (calculate it) only if it is asked to find solution in decimal places

Example 5

(i) $x^2 - 5x - 10 = 0$ Here, $a = 1, b = -5, c = -10$

Applying formula, $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$, we get

$= \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4\times1\times(-10)}}{2\times1} = \dfrac{5 \pm \sqrt{25+40}}{2} = \dfrac{5 \pm \sqrt{65}}{2}$

$= \dfrac{5 \pm 8.062}{2}$

$= \dfrac{5+8.062}{2}, \dfrac{5-8.062}{2}$

$= \dfrac{13.062}{2}, \dfrac{-3.062}{2}$

$= 6.531, -1.531$

$= 6.53, -1.53$

correct to two decimal places.

Therefore, Solution set $= \{6.53, -1.53\}$

	8.0622
8	65.00 00 00 00
	64
160	1 00
	0
1606	1 00 00
	96 36
16122	3 64 00
	3 22 44
161242	41 56 00
	32 24 84
	9 31 16

$\therefore \sqrt{65} = 8.062$

(ii) $x^2 - 3x - 9 = 0$ Here, $a = 1, b = -3, c = -9$

Applying formula, $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$, we get

$= \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4\times1\times(-9)}}{2\times1} = \dfrac{3 \pm \sqrt{9+36}}{2} = \dfrac{3 \pm \sqrt{45}}{2}$

$= \dfrac{3 \pm 3\sqrt{5}}{2}$

$= \frac{3}{2}(1+\sqrt{5}), \frac{3}{2}(1-\sqrt{5})$

$= \frac{3}{2}(1+2.236), \frac{3}{2}(1-2.236)$

$= \frac{3}{2} \times 3.236, \frac{3}{2} \times (-1.236)$

$= 4.854, -1.854$

$= 4.85, -1.85$ correct to two decimal places

	2.2360
2	5.00 00 00 00
	4
42	100
	84
443	1600
	1329
4466	27100
	26796
44720	30400
	0
	30400

Therefore solution set $= \{4.85, -1.85\}$

Example 6

(i) $\sqrt{5x+6} = x$ Squaring both sides, we get

$5x+6 = x^2 \Rightarrow x^2 - 5x - 6 = 0$

Here, $a = 1, b = -5, c = -6$

Applying formula, $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4\times 1 (-6)}}{2\times 1}$

$= \dfrac{5 \pm \sqrt{25+24}}{2} = \dfrac{5 \pm \sqrt{49}}{2} = \dfrac{5 \pm 7}{2} = \dfrac{5+7}{2}, \dfrac{5-7}{2}$

$= \frac{12}{2}, \frac{-2}{2} = 6, -1$ Here, $x = -1$ is not satisfing given equation. So, $x = 6$

(ii) $\sqrt{x} + 2x = 1 \Rightarrow \sqrt{x} = 1 - 2x$ Squaring both sides

$x = (1-2x)^2 \Rightarrow x = 1 + 4x^2 - 4x$

$\Rightarrow 4x^2 - 5x + 1 = 0$ Here $a = 4, b = -5, c = 1$

Applying formula, $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\Rightarrow x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4\times 4\times 1}}{2\times 4} = \dfrac{5 \pm \sqrt{25-16}}{8}$

$= \dfrac{5 \pm \sqrt{9}}{8} = \dfrac{5 \pm 3}{8} = \dfrac{5+3}{8}, \dfrac{5-3}{8} = \dfrac{8}{8}, \dfrac{2}{8}$

$= 1, \frac{1}{4}$ Here, $x = 1$ is not satisfying given equation, So, $x = \frac{1}{4}$

(ii) $2x^2 + \sqrt{7}x - 7 = 0$

Here, $a = 2$, $b = \sqrt{7}$, $c = -7$

Applying the formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we get

$$x = \frac{-\sqrt{7} \pm \sqrt{(\sqrt{7})^2 - 4\times 2\times(-7)}}{2\times 2} = \frac{-\sqrt{7} \pm \sqrt{63}}{4}$$

$$= \frac{-\sqrt{7} \pm 3\sqrt{7}}{4} = \frac{-\sqrt{7}+3\sqrt{7}}{4}, \frac{-\sqrt{7}-3\sqrt{7}}{4} = \frac{2\sqrt{7}}{4}, \frac{-4\sqrt{7}}{4}$$

$$= \frac{\sqrt{7}}{2}, -\sqrt{7}$$

$$= \frac{2.646}{2}, -2.646$$

$$= 1.323, -2.646$$

Solution Set $= \{1.323, -2.646\}$

Two decimal place
$\{1.32, -2.65\}$

Two significant figures
$\{1.3, -2.6\}$

	2.6457
2	$7.\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}$
	4
46	300
	276
524	2400
	2096
5285	30400
	26425
52907	397500
	370349
	27151

$\therefore \sqrt{7} = 2.646$

Important Instructions :-

1) Use mathematical tables to calculate the value of square root.
2) Calculate the value of square root only if it is asked to find the answer in decimal places or significant figure.
3) It is compulsory to find the value of square root till 4 significant figure. e.g. $\sqrt{85} = 9.219$

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NATURE OF THE ROOTS

DISCRIMINANT	NATURE OF ROOTS
$b^2 - 4ac = 0$	real, rational, equal
$b^2 - 4ac > 0$, perfect square	real, rational, unequal
$b^2 - 4ac > 0$, not perfect square	real, irrational, unequal
$b^2 - 4ac < 0$	no real roots