Tender Heart High School, Sector 33B, Chd.

Class 10, Mathematics

Date: 25.11.2024

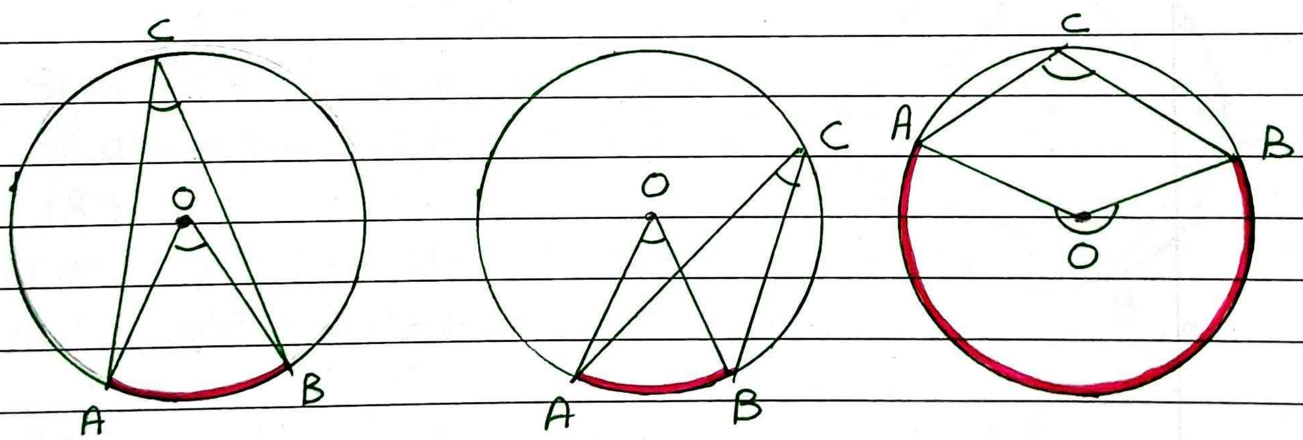
Chapter-18,19 Circles

→ A circle is a closed plane figure consisting of all those points of the plane which are at a constant distance from a fixed point in that plane.

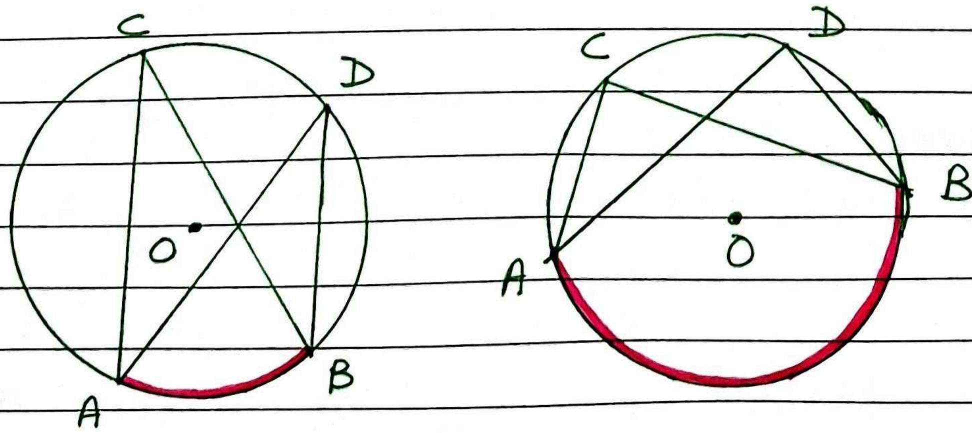
→ The fixed point is called its centre and the constant distance is called its radius.

Angle properties of circles

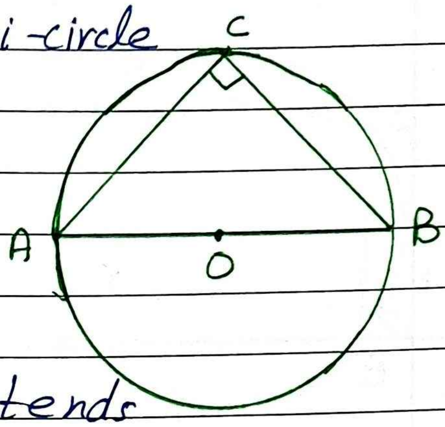
Theorem 1 The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle. i.e. $\angle AOB = 2\angle ACB$



Theorem 2 Angle in the same segment of a circle are equal i.e. $\angle ACB = \angle ADB$



Theorem 3:- Angle in a semi-circle is a right angle.
i.e. $\angle ACB = 90^\circ$
where AB is a diameter

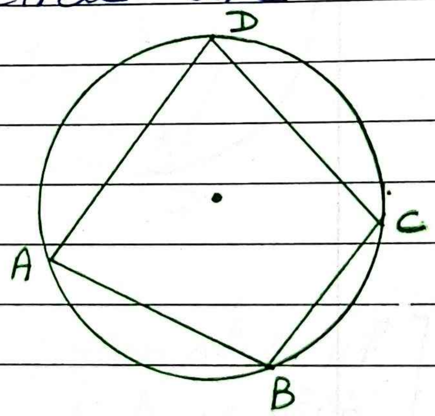


Th 4:- Converse of Theorem 3
If an arc of a circle subtends a right angle at any point on the remaining part of the circle, then the arc is a semi-circle.

Theorem 5:- The opposite angles of a quadrilateral incribed in a circle are supplementary.

OR

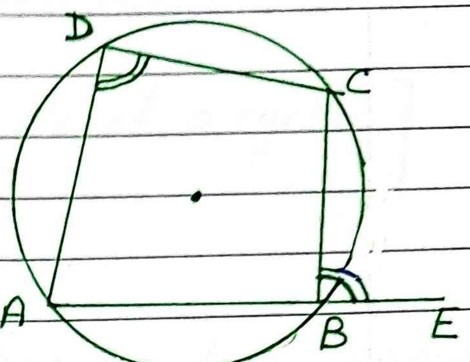
Sum of the opposite angles of a cyclic quadrilateral is 180°
i.e. $\angle ADC + \angle ABC = 180^\circ$
and $\angle BAD + \angle BCD = 180^\circ$



→ A quadrilateral is said to be a cyclic quadrilateral if there is a circle passing through all its four vertices.

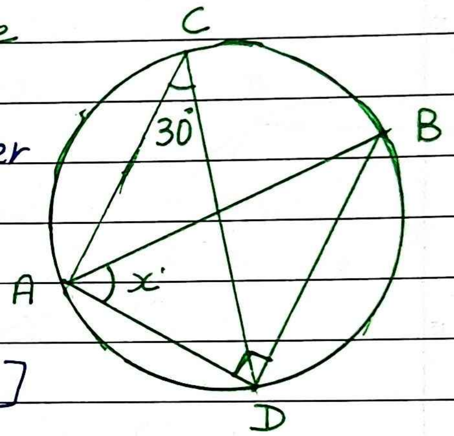
Th 6: Converse of Theorem 5
If a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.
i.e. If $\angle ADC + \angle ABC = 180^\circ$
then ABCD is a cyclic quadrilateral.

Theorem 7 The exterior angle of a cyclic quad. is equal to the interior opposite angle.
i.e. $\angle CBE = \angle ADC$



Examples:-

Example 1:- In the given circle with diameter AB, find the value of x.

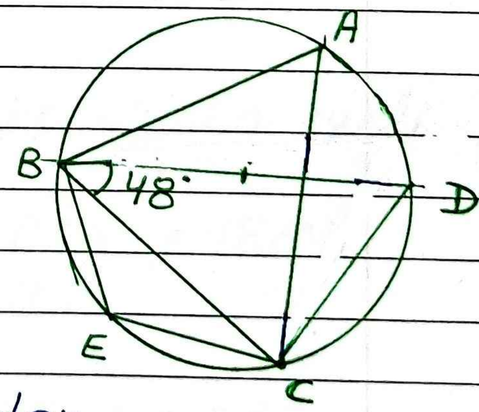


Solution:- Since, AB is diameter
$\angle ADB = 90°$ [angle in a semi-circle]

$\angle ABD = \angle ACD = 30°$
[$\angle$'s in same segment]

In ΔADB, we have
$\angle BAD + \angle ADB + \angle ABD = 180°$ [Sum of $\angle$s of a Δ is 180°]
$\Rightarrow x + 90 + 30 = 180$
$\Rightarrow x = 180 - 120 = 60$
Hence, $x = 60°$

Example 2:- In the given figure, BD is a diameter of the circle. If $\angle DBC = 48°$, find (i) $\angle BDC$ (ii) $\angle BEC$ (iii) $\angle BAC$



Solution:- (i) As BD is diameter,
$\angle BCD = 90°$ [angle in semicircle $= 90°$]
$\therefore \angle BDC = 180° - (48° + 90°) = 42°$
$\Rightarrow \angle BDC = 42°$ [sum of angles of a $\Delta = 180°$]

(ii) As BECD is a cyclic quadrilateral,

$\angle BEC + \angle BDC = 180^\circ$ [sum of opp. $\angle$'s of a cyclic quad. $= 180^\circ$]

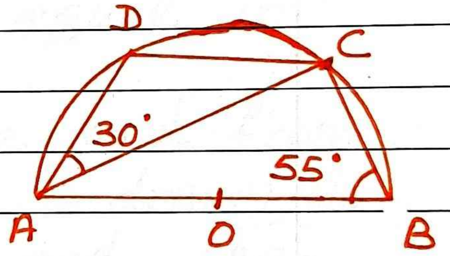
$\Rightarrow \angle BEC + 42^\circ = 180^\circ$

$\Rightarrow \angle BEC = 138^\circ$

(iii) $\angle BAC = \angle BDC = 42^\circ$ [angles in the same segment of a circle]

$\Rightarrow \angle BAC = 42^\circ$ [$\because \angle BDC = 42^\circ$]

Example 3 In the adjoining figure, C and D are points on a semi-circle described on AB as diameter. If $\angle ABC = 55^\circ$, $\angle CAD = 30^\circ$. Calculate $\angle BAC$ and $\angle ACD$.



Solution:- $\angle ACB = 90^\circ$ [Angle in a semicircle]

In ΔABC, we have

$\angle BAC + \angle ACB + \angle ABC = 180^\circ$ [Sum of the $\angle$'s of a triangle]

$\Rightarrow \angle BAC + 90^\circ + 55 = 180^\circ$

$\Rightarrow \angle BAC = 35^\circ$

Now, ABCD is a cyclic quadrilateral

$\therefore \angle BAD + \angle BCD = 180^\circ$

[Sum of the opp. $\angle$'s of a cyclic quad.]

$\Rightarrow (\angle BAC + \angle CAD) + (\angle ACB + \angle ACD) = 180^\circ$

$\Rightarrow 35^\circ + 30^\circ + 90^\circ + \angle ACD = 180^\circ$

$\Rightarrow \angle ACD = 180 - 155 = 25^\circ$

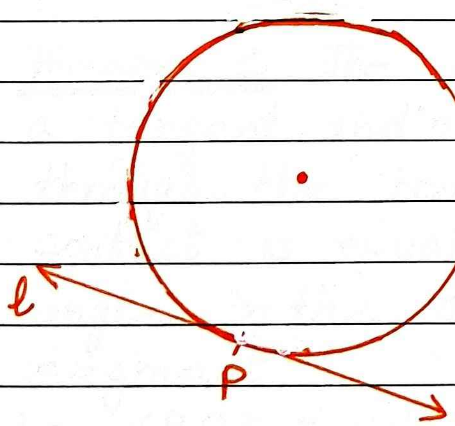
Hence, $\angle BAC = 35^\circ$ and $\angle ACD = 25^\circ$

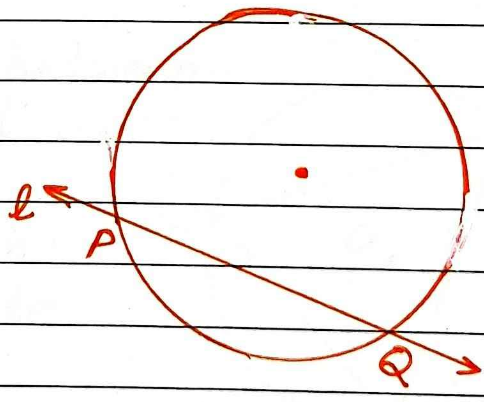
Class 10, Mathematics Ms. Reena

Chapter - 19

Tangent and secant to a circle

1) The tangent at any point of a circle and the radius through the point are perpendicular to each other.
2) From any point outside a circle two tangents can be drawn and they are equal in length.
3) If a line touches a circle and from the point of contact, a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.





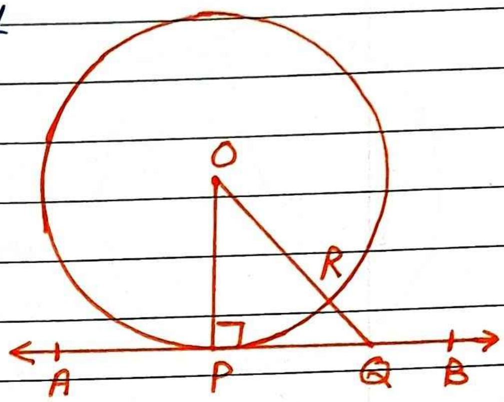
→ P is tangent to a circle

→ One point on circle

→ P and Q ie. PQ is secant to a circle

→ Two points on circle.

Theorem 1 The tangent at any point of a circle and the radius through the point are perpendicular to each other.
i.e. $OP \perp AB$

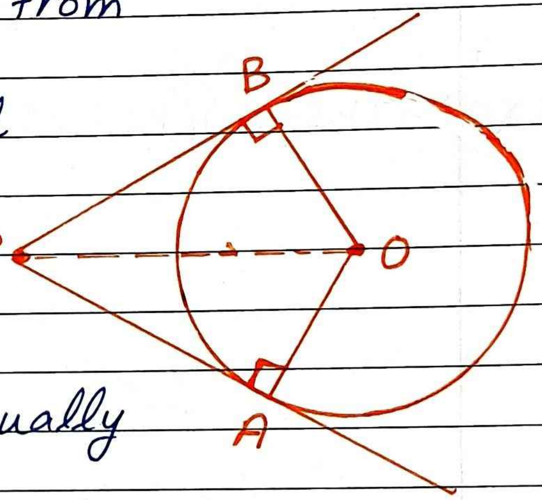


Theorem 2 If two tangents are drawn to a circle from an exterior point, then

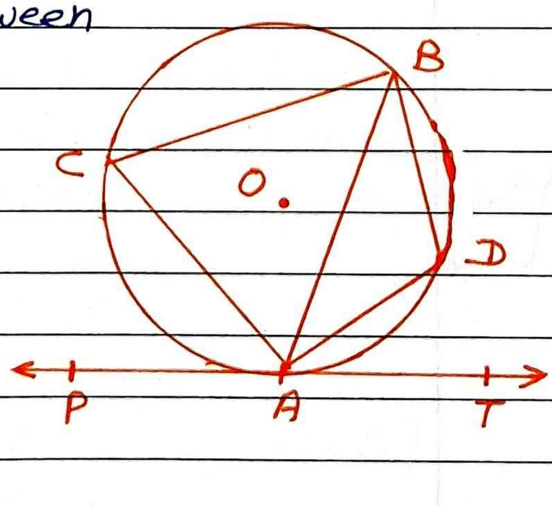
(i) the tangents are equal in length. i.e. $PA = PB$

(ii) the tangents subtend equal angles at the centre. i.e. $\angle AOP = \angle BOP$

(iii) the tangents are equally inclined to the line joining the point and the centre of the circle. i.e. $\angle APO = \angle BPO$

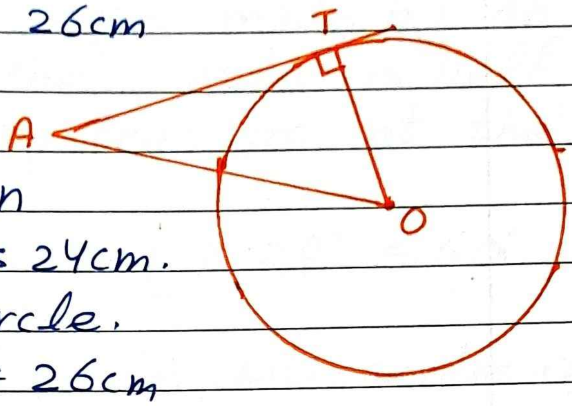


Theorem 3 The angle between a tangent and a chord through the point of contact is equal to an angle in the alternate segment.
i.e. $\angle BAT = \angle ACB$
and $\angle BAP = \angle ADB$



The chord AB divides the circle into two segments, namely ADB and BCA, called the alternate segment.

Example 1:- A point A is 26cm away from the centre of a circle and the length of tangent drawn from A to the circle is 24cm. Find the radius of circle.



Solution:- Given $OA = 26cm$

$OT \perp AT$ and $AT = 24cm$

In right ΔOTA, we have

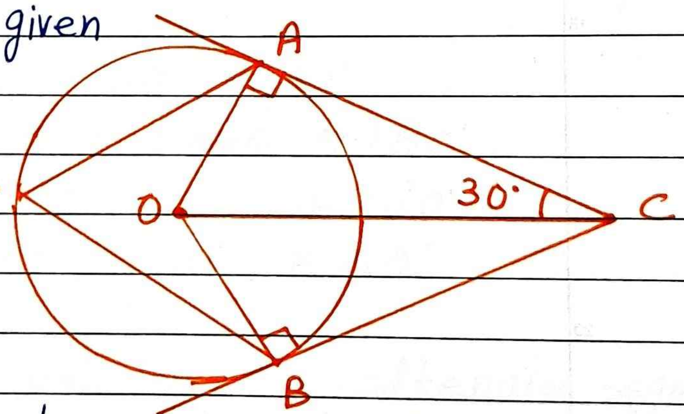
$(OT)^2 = (OA)^2 - (AT)^2$

$= (26)^2 - (24)^2 = (26+24)(26-24) = 50 \times 2$

$= 100$

$\Rightarrow OT = \sqrt{100} = 10$ cm

Example 2:- In the given figure, O is the centre of the circle. Tangents at A and B meet at C. If $\angle ACO = 30°$ find (i) $\angle BCO$ (ii) $\angle AOB$ (iii) $\angle APB$



Solution:- (i) In ΔAOC and ΔBOC, we have

$OA = OB$ [Radii of the same circle]

$AC = BC$ [Tangents are equal from a point outside a circle]

$OC = OC$ [common]

$\therefore AOC \cong \Delta BOC$

Hence, $\angle BCO = \angle ACO = 30°$ [c.p.c.t.]

(ii) clearly, $\angle OAC = \angle OBC = 90°$

$\therefore \angle AOC = 90° - \angle ACO = 90° - 30° = 60°$

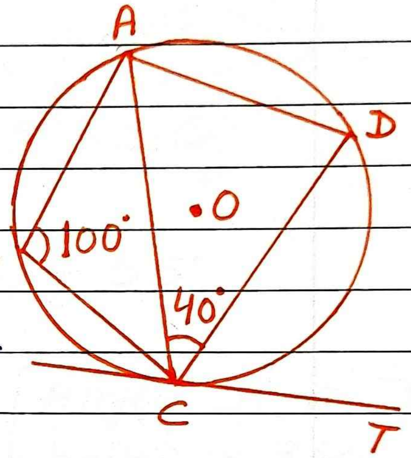
and $\angle BOC = 90° - \angle BCO = 90° - 30° = 60°$

$\therefore \angle AOB = \angle AOC + \angle BOC = 60° + 60° = 120°$

(iii) We know that the angle made by an arc at a point on the circle is half of the angle made by the arc at the centre.

$\therefore \angle APB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 120^\circ = 60^\circ$

Example 3:- In the given circle with centre O, $\angle ABC = 100^\circ$, $\angle ACD = 40^\circ$ and CT is a tangent to the circle at C. Find $\angle ADC$ and $\angle DCT$.



Solution:- We know that the opposite angles of a cyclic quadrilateral are supplementary.

$\therefore \angle ABC + \angle ADC = 180^\circ$

$\Rightarrow 100^\circ + \angle ADC = 180^\circ$

$\Rightarrow \angle ADC = 180^\circ - 100^\circ = 80^\circ$

In ΔADC, $\angle ACD + \angle ADC + \angle DAC = 180^\circ$

$\Rightarrow 40^\circ + 80^\circ + \angle DAC = 180^\circ$

$\Rightarrow \angle DAC = 180^\circ - 120^\circ = 60^\circ$

Now, $\angle DCT = \angle CAD = 60^\circ$

[Angles in the altenate segment]

Hence, $\angle DCT = 60^\circ$