TENDER HEART HIGH SCHOOL, SECTOR-33 B, CHD.

Class : 10th
Subject : Mathematics

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GEOMETRIC PROGRESSION (G.P.)

→ A list of numbers in which each term is obtained by multiplying its preceding term by a fixed (non-zero) number, except the first term, is called a geometric progression. Thus, a list of numbers forms a G.P. if and only if the ratio of any term to its preceding term is constant.

→ This fixed number is denoted by 'r'

If, $a_1, a_2, a_3, a_4, \ldots$ are in G.P.

Then, common ratio $(r) = \frac{a_2}{a_1}$ or $\frac{a_3}{a_2} \ldots \frac{a_{n+1}}{a_n}$

→ The General form of G.P. is $a, ar, ar^2, \ldots$

$\Rightarrow a_n = ar^{n-1}$

→ If the last term of a G.P. consisting of 'n' terms is denoted by l, then $l = ar^{n-1}$

Example 1:- Determine whether the given sequence is geometric or not. $-1, 6, -36, 216, \ldots$
$a_1, a_2, a_3, a_4, \ldots$

Here, first term $(a) = -1$

and common ratio $= \frac{a_2}{a_1} = \frac{6}{-1} = -6$, $\frac{a_3}{a_2} = \frac{-36}{6} = -6$

$\frac{a_4}{a_3} = \frac{216}{-36} = -6, \ldots$

Thus, common ratio here is -6 which is same in all cases. Hence, it is in G.P.

Note : For A.P., we find 'd' $= a_2 - a_1$

For G.P., we find 'r' $= \frac{a_2}{a_1}$

where, $a_1\ a_2, a_3, a_4 \ldots$ is a series

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Example 2:- Find the n^{th} and the 15^{th} term of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + --- + ---$

Solution: Here, first term 'a' = 1 and common ratio $= \frac{-1/2}{1} = -\frac{1}{2}$

Hence, n^{th} term, $a_n = ar^{n-1} = 1 \times \left(-\frac{1}{2}\right)^{n-1}$
$= (-1)^{n-1}(2)^{1-n}$

and $a_{15} = 1 \times \left(-\frac{1}{2}\right)^{15-1}$
$= (-1)^{15-1} \times 2^{1-15} = 2^{-14}$

Example 3:- The 4th term of a G.P. is 16 and the 7th term is 128. Find the first term and the common ratio of the G.P.

Solution:- Let 'a' be the first term and 'r' be the common ratio of the G.P.

Given, $a_4 = 16 \Rightarrow ar^{4-1} = 16 \Rightarrow ar^3 = 16$ ---(i)
and $a_7 = 128 \Rightarrow ar^{7-1} = 128 \Rightarrow ar^6 = 128$ ---(ii)

Dividing equation (ii) by (i), we get
$\frac{ar^6}{ar^3} = \frac{128}{16} \Rightarrow r^3 = 8 \Rightarrow r = 2$

From (i), $a \times 2^3 = 16 \Rightarrow 8a = 16 \Rightarrow a = 2$

Hence, first term = 2 and common ratio is 2.

Note :- 1) If x, y, z are in A.P.
then $2y = x + z$
But if x, y, z are in G.P.
then, $y^2 = xz$

2)(i) 3 terms of G.P. are $\frac{a}{r}, a, ar$

(ii) 4 terms of G.P. are $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

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Question 1 $\sqrt{3}, 3, 3\sqrt{3}, 9, 9\sqrt{3}, \ldots$

Here, first term $= \sqrt{3}$, common ratio i.e. $r = \frac{3}{\sqrt{3}} \rightarrow \frac{a_2}{a_1}$

$= \frac{3\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \sqrt{3}$

Also, $r = \frac{a_3}{a_2} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

or $r = \frac{a_4}{a_3} = \frac{9}{3\sqrt{3}} = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

or $r = \frac{a_5}{a_4} = \frac{9\sqrt{3}}{9} = \sqrt{3}$

Since, common ratio i.e r is same in all cases so, given sequence form a G.P.

Question 2 $\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}$ Here, $a = \sqrt{2}$, $r = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{2}$

Now, $a_n = ar^{n-1} \Rightarrow a_8 = \sqrt{2}\left(\frac{1}{2}\right)^{8-1}$

$\Rightarrow a_8 = \sqrt{2}\left(\frac{1}{2}\right)^7 = 2^{\frac{1}{2}} \cdot 2^{-7} = 2^{\frac{1}{2}-7} = 2^{-\frac{13}{2}}$

OR $a_8 = \frac{\sqrt{2}}{2^7} = \frac{\sqrt{2}}{128}$ or $\frac{1}{64\sqrt{2}}$

Question 3 $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \ldots$ is $-\frac{1}{243}$?

Here, $a = 1$, $r = \frac{-1/3}{1} = \frac{-1}{3}$, $a_n = \frac{-1}{243}$

$a_n = ar^{n-1} \Rightarrow \frac{-1}{243} = 1\left(\frac{-1}{3}\right)^{n-1}$

$\Rightarrow \left(\frac{-1}{3}\right)^5 = \left(\frac{-1}{3}\right)^{n-1} \Rightarrow 5 = n-1 \Rightarrow n = 6$

So 6 th term is $\frac{-1}{243}$

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Question 4

$-\frac{2}{7}, p, -\frac{7}{2}$ form a G.P. ?

We know that when, a, b, c are in G.P.

Then, $b^2 = ac$

$\Rightarrow p^2 = -\frac{2}{7} \times -\frac{7}{2} \Rightarrow p^2 = 1 \Rightarrow p = \pm 1$

Question 5

$a_8 = 192,\ r = 2,\ n = 8,\ a_{12} = ?$

$a_n = ar^{n-1}$

$\Rightarrow 192 = a(2)^{8-1} \Rightarrow \frac{192}{2^7} = a$

$\Rightarrow a = \frac{192}{128} = \frac{3}{2}$

Now, $a_{12} = ar^{12-1} \Rightarrow a_{12} = \frac{3}{2}(2)^{12-1}$

$\Rightarrow a_{12} = \frac{3}{2}(2)^{11} = 3 \times 2^{10} = 3 \times 1024 = 3072$

Question 6-

Given 162, 54, 18, 6, ----

Here, $a = 162,\ r = \frac{54}{162} = \frac{1}{3}$

So, $a_n = 162\left(\frac{1}{3}\right)^{n-1}$

other, G.P. is. $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}, \frac{2}{3}$, ----

Here, $a = \frac{2}{81},\ r = \frac{2}{27} \div \frac{2}{81} = 3$

So, $a_n = \frac{2}{81}(3)^{n-1}$

Since, nth term of G.P. are equal

$\Rightarrow 162\left(\frac{1}{3}\right)^{n-1} = \frac{2}{81}(3)^{n-1}$

$\Rightarrow 162 \times \frac{81}{2} = (3)^{n-1} \div \left(\frac{1}{3}\right)^{n-1}$

$\Rightarrow 81 \times 81 = (3)^{n-1} \times (3)^{n-1} \Rightarrow 3^8 = 3^{2(n-1)}$

$\Rightarrow 8 = 2(n-1) \Rightarrow 4 = n-1 \Rightarrow n = 5$

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Sum of 'n' terms of a Geometric Progression

→ If 'a' is the first term, 'r' the common ratio and S_n the sum of n terms of the G.P., then

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1} \quad \text{------ (i)}$$

$$\Rightarrow rS_n = ar + ar^2 + ar^3 + \cdots + ar^{n-1} + ar^n \quad \text{--(ii)}$$

Now subtracting equation (ii) from equation (i), we have

$$S_n - rS_n = a - ar^n$$

$$\Rightarrow S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{1-r} \; ; \; r \neq 1 \; (r < 1)$$

$$\text{or} \quad \frac{a(r^n-1)}{r-1} \quad (r > 1)$$

Example 1:- Find the sum of the G.P. 4, 2, 1, --- to 10 terms.

Solution:- Here, $a = 4$, $r = \frac{2}{4} = \frac{1}{2}$, $n = 10$

We have, $S_n = \frac{a(1-r^n)}{1-r}$ (Here $r = \frac{1}{2} < 1$)

Therefore, $S_{10} = \frac{4\left[1-\left(\frac{1}{2}\right)^{10}\right]}{1-\frac{1}{2}} = \frac{4\left(1-\frac{1}{1024}\right)}{\frac{1}{2}}$

$$= \frac{2 \times 4 \times 1023}{1024} = \frac{1023}{128}$$

Example 2:- Given a G.P. with $a = 729$, 7^{th} term $= 64$ determine S_7

Solution:- $a_n = ar^{n-1} \Rightarrow a_7 = 64 = ar^{7-1}$

$$\Rightarrow 729r^6 = 64 \Rightarrow r^6 = \frac{64}{729} = \left(\frac{2}{3}\right)^6$$

$$\Rightarrow r^6 = \frac{2}{3} \text{ or } -\frac{2}{3}$$

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Example 3:- Find the number of terms of a G.P. if $a_1 = 3$, $a_n = 96$ and $S_n = 189$

Solution:- $a_n = ar^{n-1}$

$\Rightarrow 96 = 3r^{n-1} \Rightarrow r^{n-1} = 32$ ----- (i)

Now, $S_n = \dfrac{a(r^n - 1)}{r-1} \Rightarrow 189 = \dfrac{3(r^n - 1)}{r-1}$

$\Rightarrow 63(r-1) = r^n - 1 \Rightarrow 63r - 62 = r.r^{n-1}$

$\Rightarrow 63r - 62 = r \times 32$ [using (i)]

$\Rightarrow 31r = 62 \Rightarrow r = 2$

Substituting this value of r in (i), we get

$2^{n-1} = 32 \Rightarrow 2^{n-1} = 2^5 \Rightarrow n - 1 = 5$

$\Rightarrow n = 6$ Hence, the number of terms = 6

Example 4:- Find the sum of first 'n' terms of the series $5 + 55 + 555 + ---$

Solution:- Let S be the required sum, then

$S = 5 + 55 + 555 + -----$ upto n terms

$= 5(1 + 11 + 111 + ----$ upto n terms)

$= \frac{5}{9}(9 + 99 + 999 + ----$ upto n terms.

$= \frac{5}{9}\left[(10-1) + (100-1) + (1000-1) + ---\text{ upto } n \text{ terms}\right]$

$= \frac{5}{9}\left[(10 + 100 + 1000 + -----) - (1 + 1 + 1 - -----)\right]$

$= \frac{5}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right] = \frac{5}{9}\left[\frac{10^{n+1} - 10 - 9n}{9}\right]$

$= \frac{5}{81}\left(10^{n+1} - 10 - 9n\right)$

Summary of A.P. and G.P.

Arithmetic Progression

* $a, a+d, a+2d, a+3d, \ldots$
* $a_n = a + (n-1)d$

where, a = first term

d = common difference

* $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$
* $a_n = S_n - S_{n-1}$
* Middle term

n (odd) $\rightarrow \left(\frac{n+1}{2}\right)^{th}$ term

n (even) $\rightarrow \left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$

* If a, b, c in A.P.

Then, $2b = a + c$

Geometric Progression

* $a, ar, ar^2, ar^3, \ldots$
* $a_n = ar^{n-1}$

where, a = first term

r = common ratio

* $S_n = \frac{a(1-r^n)}{1-r}$ $\quad r \neq 1$

or $a\left(\frac{r^n - 1}{r-1}\right)$

If $r = 1$, then

$S_n = a + a + a + \cdots = na$

* If a, b, c in G.P.

Then, $b^2 = ac$